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Portfolio performance gauging in discrete time using a Luenberger productivity indicator ☆

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ABSTRACT

This paper proposes a pragmatic, discrete time indicator to gauge the performance of portfolios over time. Integrating the shortage function (Luenberger, 1995) into a Luenberger portfolio productivity indicator (Chambers, 2002), this study estimates the changes in the relative positions of portfolios with respect to the traditional Markowitz mean-variance efficient frontier, as well as the eventual shifts of this frontier over time. Based on the analysis of local changes relative to these mean-variance and higher moment (in casu, mean-variance-skewness and mean-variance-skewness-kurtosis) frontiers, this methodology allows to neatly separate between on the one hand performance changes due to portfolio strategies and on the other hand performance changes due to the market evolution. This methodology is empirically illustrated using a mimicking portfolio approach (Fama and French, 1996, 1997) using US monthly data from January 1931 to August 2007.

1. Introduction

Since Markowitz (1952) foundational work, every investor knows that to gauge the performance of portfolio management risk must be considered in addition to return levels. This mean-variance (MV) dual objective of maximizing returns and minimizing risks turns performance evaluation into a controversial task. Indeed, no method that is currently available in the literature seems to be universally approved. There is an ever growing literature on this topic in traditional investment contexts (for surveys, see, e.g., Cuthbertson et al. (2008) on mutual funds), as well as in the specific context of hedge fund management (for instance, Eling and Schuhmacher (2007)), and even a meta-literature criticizing these methods as well (see, for example, Bacon (2008)).

Performance appraisal is linked to the theory of optimal investment choices, i.e., to the ability of investors to manage assets so as to maximize a utility function (based on a set of various moments characterizing the portfolios' return distributions). In other words, performance evaluation analyzes the efficiency of an investment at least in terms of a traditional return–risk relationship. It is often assumed that all investors have similar behaviors towards these dimensions (representative agent paradigm). The risk characteristics in the utility function depend upon various parameters like investor's objectives, preferences, time horizon,. . .. These simplifications are acceptable in cases where aggregate results suffice, but these are simply problematic in other cases. The methodology proposed in this paper allows for heterogeneity among investors and therefore answers quite a few of these issues.

Several models for portfolio selection based on higher-order moments have been developed in the literature (e.g., Philippatos, 1979). However, none of these procedures has managed to obtain widespread acceptance. For instance, Lai (1991) determines mean-variance-skewness (MVS) optimal portfolios via a multi-objective programming approach (see Chunhachinda et al. (1997) for an empirical application). However, in this contribution we explore general moment portfolio models (see Briec et al., 2007; Briec and Kerstens, in press), while explicitly limiting ourselves in the empirical part to the first four moments traditionally considered

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in the financial literature. To be explicit, we consider MV, MVS and mean-variance-skewness-kurtosis (MVSK) models, the latter to account for the observed fat tails that prove to be important in capital markets. On the one hand, while the MV approach is still a popular reference for practitioners and academics alike, its restrictive nature may lead to erroneous weights in portfolio selection. While some proposals are around allowing investors to maximize a utility function including higher moments (e.g., Jondeau and Rockinger, 2003), the empirical evidence provides mixed support at best. Nevertheless, enlarging the classical framework with MVS and MVSK models is a potentially interesting improvement for fund managers. On the other hand, the method developed in this research can be easily extended to consider even higher moments, but at an increasingly important computational cost.

Recently, a new approach has been proposed in the investment literature by Cantaluppi and Hug (2000) that directly measures the performance of a portfolio by reference to its maximum potential on the (ex-ante or ex-post) portfolio frontier. Their proposal is in fact intimately related to some explicit efficiency measures transposed from production theory into the context of portfolio benchmarking by Morey and Morey (1999) in the operations research literature. Informally speaking, their first measure computes the maximum mean return expansion while the risk is fixed at its current level, while an alternative risk contraction function measures the maximum proportionate reduction of risk while fixing the mean return level.[1]

These explicit efficiency approaches are generalized by Briec et al. (2004) who integrate the shortage function (Luenberger, 1995) as an efficiency measure into the MV model and also develop a dual framework to assess the degree of satisfaction of investors preferences. Similar to developments in other fields, this leads to a decomposition of portfolio performance into allocative and portfolio efficiency. The advantage is that this shortage function is compatible with general investor preferences and that it can be extended to higher dimensional spaces (e.g., MVS space Briec et al. (2007) or even higher-order models Briec and Kerstens (in press)).

This paper tackles the problem of tracing the performance of portfolios in discrete time with respect to the ever changing portfolio frontiers by borrowing from recent developments in the theory of productivity indices (see Diewert (2005) for a review). Employing the shortage function, a Luenberger portfolio productivity indicator (Chambers, 2002) is introduced that allows for the estimation of the relative positions of portfolios with respect to changes in the efficient frontier, and that offers an accurate local measure of the eventual shifts of this frontier over time. The proposed methodology for fund performance appraisal in discrete time is therefore founded in a well-established theoretical framework. This Luenberger portfolio productivity indicator and its decomposition provide an excellent measurement tool to reconsider the traditional performance attribution question: what is the individual contribution of fund managers to portfolio performance and what is due to changes in the financial market. To the best of our knowledge, this contribution is the first to integrate recent developments in index theory into the portfolio performance evaluation framework.

By positioning ourselves into an extended Markowitz-like approach, we do not impose the much stronger assumptions usually maintained in the CAPM context. While the advantage of using a frontier as a benchmarking tool may be obvious, one should be aware of the fundamental relative nature of this frontier with respect to the selected asset universe.[2] Thus, we do not claim our method is a new test of the efficiency of a given portfolio relative to an equilibrium theory of financial markets as proposed in the more traditional literature (e.g., Gibbons et al., 1989). We rather propose a method to identify ex-ante or ex-post improvements that can be attributed to funds managers when they optimize their positions relative to a limited asset universe. Indeed, one should notice that the qualification of efficiency is conditioned on its timing. Ex-post efficiency refers to an appraisal of performance once returns (consequently, all moments) are known, while ex-ante efficiency refers to a similar task based on expected returns. Obviously, prospective benchmarking is surrounded with a multitude of problems related to the fundamental uncertainties in the data. This requires special attention in terms of statistical inference on the eventual efficiency status of ex-ante decisions regarding the ex-post results (see Markowitz, 1952).

The next section is devoted to a brief presentation of the relevant literature concerning portfolio performance evaluation and the more recently introduced efficiency measures operating relative to the portfolio frontier. Section 3 introduces the basic theoretical building blocks for the analysis. In particular, it introduces the shortage function as proposed by Luenberger (1992) and studies its axiomatic properties. Thereafter, we present the Luenberger portfolio productivity indicator and its decomposition. Section 4 deals with some technical and strategic aspects of the empirical procedures and discusses the choice of data set. We comment upon the empirical results in Section 5. Conclusions and issues for future work are summarized in the final section.

2. Performance measurement in investment: A brief review

2.1. Traditional performance measures

An enormous literature on portfolio performance evaluation builds upon the initial work of Markowitz (1952) and the founders of modern portfolio theory with the development of asset pricing theories (e.g., the CAPM). During these early years, performance appraisal evolved from total-risk foundations (e.g., the standard deviation or variance of returns) to performance indexes where the returns in excess of the risk-free rate are matched with some risk measure. Two classics are the Sharpe ratio and the Treynor ratio, which gauge performance without any benchmark. These and more recent developments of portfolio performances gauges are nicely surveyed by Bacon (2008).

This tradition has received a wide variety of criticisms because of the supposed weaknesses of the underlying equilibrium models on which performance indicators were built and the implicit assumption that financial asset returns are normally, independently and identically distributed, among others.[3]

The first series of objections touches upon several issues. One is the irrelevance of unconditional performance evaluation: investors are supposed to form expectations about returns irrespective of their expectations over the states of the economy, which may lead to various distortions in performance levels or stability. A second problem is linked to the choice of benchmark to gauge portfolio performance, especially when funds have different management styles. When the reference point is inappropriate, then the measure is biased (see Grinblatt and Titman, 1994). A third series of problems with the underlying equilibrium models (recognized ever since Jensen (1972)) comes from the non-stability of risk-free rates or the volatility of betas. In these cases, performance evaluation is biased because equilibrium returns are misevaluated or simply because a constant beta is irrelevant.

Another major concern is the non-Gaussian nature of stock returns due to dynamic trading strategies (for instance, hedge funds are a case in point: see Darolles and Gourieroux (2010)). Problematic

[1] Cantaluppi and Hug (2000) talk similarly about return loss and surplus risk.

[2] Obviously, all empirical work within a CAPM framework refers as a matter of fact to geographically limited parts of a potentially universal financial market.

[3] See, for instance, Fama and French (2004) on the debate surrounding CAPM.

here is the underestimation of risk in performance appraisal. With asymmetric or fat tailed distributions, performance gauging must account for higher-order moments (skewness, kurtosis or even beyond: see Ang and Chua (1979)) or lower partial moments (e.g., the Sortino ratio is based on a target return and semi-variance). More recently, various other proposals are formulated: some of these derive from VaR (Gregoriou and Gueyie, 2003), some are extensions of the Sharpe (Zakamouline and Koekebakker, 2009) or Sortino (Kaplan and Knowles, 2004) ratios, while still others propose even more generalized methods such as the Omega measure (see Kazemi et al., 2004).

Many of these traditional performance measures are associated with a prominent question in the investment industry, namely performance attribution. While in blatant contradiction with CAPM theory, performance appraisal is linked to *stock picking* and *market timing*. Indeed, the investment industry is always looking for tools to trace good fund managers that can exploit market anomalies and that can pick stocks in the market to obtain an alpha that is significantly different from zero and manage their portfolios' betas dynamically.

Summing up, the standard approaches to investment performance appraisal appear unsatisfactory with respect to at least three generic shortcomings: (i) these yield under- or over-estimations because of the selection of an inappropriate benchmark or equilibrium models for expected returns, (ii) these are biased due to the non-normal nature of return distributions or unknown utility functions for investors when higher moments have to be considered, and (iii) these are unstable because of the dependency of the measure with the time-frame in which it is computed. One could also add that these measures usually rely upon other strong assumptions, such as the uniqueness of investor's preferences. We now turn to the rather recent frontier-based measures that may be a solution for some of these shortcomings.

2.2. Frontier-based efficiency measures

Frontier-based measures of fund performance have gained some limited popularity since the late nineties. One of the seminal articles in the finance literature is the work of Cantaluppi and Hug (2000) who propose an efficiency ratio in relation to the MV efficient frontier.[4] In fact, their contribution is similar to the one of Morey and Morey (1999) in the operations research literature. In their search for a more universal approach to portfolio performance measurement, Cantaluppi and Hug (2000) contest the relative nature of most current proposals that define performance with respect to some other, supposedly relevant, portfolio or index. Instead, they suggest looking for the maximum performance that could have been achieved by a given portfolio relative to a relevant portfolio frontier, i.e., a frontier resulting from a particular choice of investment universe and satisfying any additional constraints imposed on the investor.

Basically, it is a matter of utilizing the traditional ex-ante computation of optimal portfolios in an ex-post fashion. Ex-ante, one first selects the investment universe; then one determines the investment horizon with corresponding estimates for future returns, risks, and correlations for the asset universe; and finally one computes an efficient frontier based on these estimates and the investment restrictions. This same process can be executed ex-post to benchmark portfolios: computations are then simply performed with historical rather than expected values. Since a portfolio manager that ex-ante would have perfect foresight could invest in a frontier optimal portfolio, the ex-post efficient frontier offers a natural benchmark for performance gauging. In this context, Cantaluppi and Hug (2000) informally present both a return loss and a surplus risk efficiency measure.

We illustrate this basic point with Fig. 1 (in the spirit of Cantaluppi and Hug (2000)) which compares the Sharpe ratio and the efficiency ratio. This figure is drawn in the mean-standard deviation space and depicts three portfolios *A*, *B*, and *C* with respect to a common portfolio frontier. Starting with the Sharpe ratio, it is clear that portfolio *C* enjoys a higher Sharpe ratio compared to portfolios *A* and *B* (i.e., the slope of the line *S*1 being greater than the slope of *S*∗), despite the fact that the latter portfolios are part of the Markowitz frontier (*EFF*1) while portfolio *C* is not. To remedy this problem, the efficiency ratio approach suggests measuring the inefficiency of portfolio *C* using either a return loss efficiency measure (vertical projection line, towards point *F*), or a surplus risk efficiency measure (horizontal projection line, towards point *E*).

To contrast existing viewpoints, we explicitly position our contribution relative to a seminal article by Gibbons et al. (1989) proposing a test of the efficiency of a given portfolio within a CAPM framework. Reconsidering Fig. 1, when a risk-free asset is available, then the portfolio frontier is a straight line (*EFF*2) with a slope τ_2, which is tangent to the so-called market portfolio at point *E*. Thus, τ_2 is the ex-post price of risk as measured within this sample. To evaluate the ex-ante efficiency of portfolio *C*, considering that it earns a risk price τ_1 (i.e., the slope of S_1), Gibbons et al. (1989) propose a test statistic based on $\phi = \left(\sqrt{1+\tau_2^2}\right)/\left(\sqrt{1+\tau_1^2}\right)$ to measure portfolio performance. The bottom line is that the slopes τ_1 and τ_2 have to be statistically different if one wants to reject the hypothesis of ex-ante efficiency for portfolio *C*, even if it is clearly situated both under *EFF*1 and *EFF*2. In other terms, ex-post efficiency can be used as an ex-ante efficiency proxy, but this raises serious statistical problems.

Notice that mathematical formulations in this contribution are expressed in terms of expected returns, while the empirical part uses historical returns for illustrative purposes. This raises the traditional ex-ante/ex-post performance appraisal issue. Two reasons justify this choice. First, in view of the efficient market hypothesis, one can view historical returns as a simplified (although weak, see for example, Elton (1999)) mechanism to generate expected return information. Another solution consists in obtaining such expected returns information from scenario analysis or from specialized firms (e.g., the I/B/E/S databases of Thomson Reuters that reflect consensus estimates). In a similar vein, we maintain the hypothesis of historical volatility stability instead of using stochastic volatility

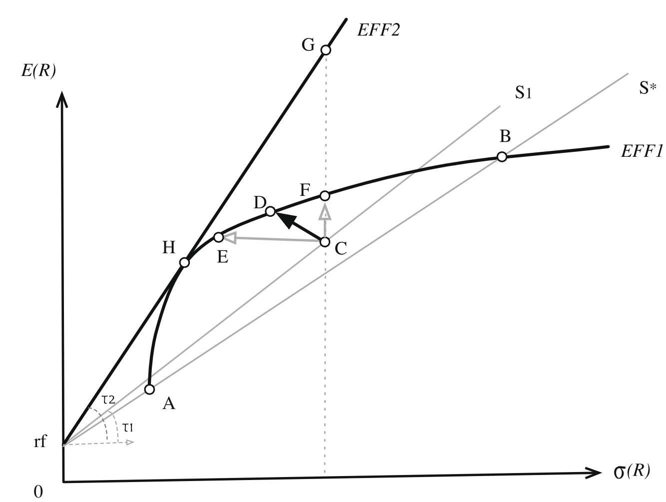

Fig. 1. Sharpe ratio vs. efficiency measures.

[4] As stated by these authors, this is not strictly speaking a new method since it has been employed by, e.g., Kandel and Stambaugh (1995) as well.

models or implied volatility derived from option pricing models. This same logic also applies to the higher moment information employed in this research. Second, this ex-ante/ex-post problem is taken into account by mixing several shortage functions based on forward and backward returns (see (6) and (7) further on).

Morey and Morey (1999) are the first to give a precise formal definition of the return loss and surplus risk efficiency measures also proposed by Cantaluppi and Hug (2000).[5] In the same vein, Briec et al. (2004) are the first to develop a link between portfolio efficiency measures and MV utility, which leads them to propose an efficiency measure that simultaneously seeks to improve return and to reduce variance of a given portfolio.[6] In Fig. 1, this leads "intuitively speaking" to the projection of portfolio C into a diagonal direction towards the Markowitz frontier. Theoretically, these contributions bring portfolio theory in line with developments in production theory and elsewhere in micro-economics, where distance functions as functional representations of choice sets are proven concepts related to efficiency measures that allow to develop dual relations with economic (e.g., MV utility) support functions.

More or less independently, a variety of authors have been transposing efficiency measures, that are related to distance functions from production theory into finance. This literature employs mathematical programming techniques to estimate non-parametric frontiers of choice sets and positions any observation with respect to the boundary of these choice sets.[7] This has sometimes been accompanied with the utilization of frontiers to rate, for instance, the performance of mutual funds along a multitude of dimensions (rather than mean and variance solely). To the best of our knowledge, the seminal article of Murthi et al. (1997) employs return as a desirable output to be increased and risk and a series of transaction costs as an input to be reduced, and measures the performance of each mutual fund with respect to a piecewise linear frontier (rather than a traditional non-linear portfolio frontier). Extensions to evaluating hedge funds have been proposed in Gregoriou et al. (2005). More recently, Glawischnig and Sommersguter-Reichmann (2010) employ a similar framework and critically compare the resulting higher-order moment efficiency measures to traditional financial indices. The same idea has been employed in the context of asset selection, whereby changes in stock performance are related to changes in productive efficiency. Preliminary results suggest that changes in productive efficiency are at least partially translated into changes in stock prices (see, e.g., Edirisinghe and Zhang (2007) or Nguyen and Swanson (2009) for recent developments).

Therefore, it is possible to state that frontier-based portfolio benchmarking methods at least partially remedy some of the generic shortcomings of traditional performance measures mentioned earlier: (i) these select an appropriate benchmark in terms of the ex-post portfolio frontier, and (ii) these can be perfectly generalized to higher moments in case of non-normal return distributions. It remains to be seen how these behave under extensive stress testing. This contribution also aims to remedy to some extent the third defect mentioned in the previous subsection, i.e., the instability of performance measures because of the dependency of these measures with respect to the time-frame in which these are computed. We resolve this at least partially by defining a portfolio productivity indicator based upon general efficiency measures that allows tracking the evolution in financial markets in discrete time. To the best of our knowledge, this is the first contribution drawing upon index theory to resolve practical portfolio benchmarking issues.

3. Static portfolio frontiers and their evolution in discrete time

3.1. Static portfolio frontiers: The shortage function as efficiency measure

To introduce some basic notation and definitions, consider the problem of selecting a portfolio from n financial assets at time period t. Let $R_{1,t},\dots,R_{n,t}$ be random returns of assets $1,\dots,n$ in period t. For each time period t, each of these assets is defined through some expected return $E[R_{i,t}]$ for $1,\dots,n$. Furthermore, returns of assets i and j are correlated, so that the variance–covariance matrix Ω_t for time period t is defined as $\Omega_{i,j,t} = Cov[R_{i,t}, R_{j,t}]$ for $i,j \in \{1,\dots,n\}$.

Notice that by adding skewness–coskewness and kurtosis–cokurtosis tensors, the extension to the MVS and MVSK frontiers is rather straightforward. Indeed, the shortage function is compatible with general investor preferences (favoring uneven moments and disliking even moments). Thus, in the MVS (MVSK) space a shortage function is capable to look simultaneously for reductions in risk (and kurtosis) and augmentations in return and skewness. In view of the familiarity of the traditional MV frontier notion and for reasons of space, the formal analysis is limited to the MV case.[8]

A portfolio $x_t = (x_{1,t},\dots,x_{n,t})$ at time period t is simply a vector of weights specified over these n financial assets that sums to unity $\left(\sum_{i=1,\dots,n} x_{i,t} = 1\right)$. If shorting is impossible, then these weights must satisfy the non-negativity conditions ($x_{i,t} \geqslant 0$). The return of portfolio x_t at time period t is given by $R_t(x_t) = \sum_{i=1,\dots,n} x_{i,t} R_{i,t}$. Therefore, the expected return of portfolio x_t is $E[R_t(x_t)] = \sum_{i=1,\dots,n} x_{i,t} E[R_{i,t}]$, and its variance is $V[R_t(x_t)] = \sum_{i,j} x_{i,t} x_{j,t} Cov[R_{i,t}, R_{j,t}]$.

The set of admissible portfolios $\Im$ can be written in general as:[9]

$$\Im = \left\{ x \in \mathbb{R}^n : \sum_{i=1,\dots,n} x_i = 1,\ x \geqslant 0 \right\}. \tag{1}$$

Following the seminal approach by Markowitz (1952), one can define at time period t the MV representation of the set $\aleph_t$ of portfolios as:

$$\aleph_t = \{(V[R_t(x_t)], E[R_t(x_t)]) : x_t \in \Im\}. \tag{2}$$

Since such a representation cannot be used for quadratic programming because the subset $\aleph_t$ is non-convex (see Briec et al., 2004), the above set is extended by defining a MV (portfolio) representation set through

$$\mathfrak{R}_t = \aleph_t + \mathbb{R}_+ \times (-\mathbb{R}_+). \tag{3}$$

Briec et al. (2004) show that it is useful to rewrite the above subset as follows:

$$\mathfrak{R}_t = \{(V', E') \in \mathbb{R}_+ \times \mathbb{R} : \exists x_t \in \Im,\ (-V', E') \leqslant (-V[R_t(x_t)], E[R_t(x_t)])\}. \tag{4}$$

[5] Even though Morey and Morey (1999) and Cantaluppi and Hug (2000) seem to be unaware of one another. Sengupta (1989) is to our knowledge the first author to transpose the idea of a return loss efficiency measure into a MV frontier context.

[6] A generalization of the same approach into MVS space is developed in Briec et al. (2007). For higher moments: see Briec and Kerstens (in press).

[7] This approach is often referred to with the moniker data envelopment analysis (DEA).

[8] The interested reader is referred to Briec et al. (2007) and Briec and Kerstens (in press) for details on the use of the shortage function relative to the MVS frontier and beyond.

[9] In this contribution, $\Im$ is time independent. However, this set of admissible portfolios can be modified to include additional constraints (e.g., transaction costs) that can be written as linear functions of asset weights: see Briec et al. (2004). Briec and Kerstens (in press) explicitly consider the cases of the availability of a risk-free asset and shorting. These additional constraints could eventually be time dependent, thereby imposing time dependency on $\Im$.

The addition of the cone is necessary for the definition of a sort of "free-disposal hull" of the MV representation of feasible portfolios and is compatible with the definition in Markowitz (1952). It is of interest to focus on the basic properties of the subset $\Re_t$ on which we define the shortage function below. Briec et al. (2004) have shown that $\Re_t$ is convex, closed and satisfies a free-disposal assumption. These properties of the representation set allow defining an efficiency measure in the context of the Markowitz portfolio theory.

Before generalizing the well-known Markowitz approach, we introduce the shortage function at time period t.[10]

Definition 3.1. The function $S_t : \Im \times \mathbb{R}^2_+ \to \mathbb{R}_+ \cup \{+\infty\}$ defined by

$$S_t(x_t; g_t) = \sup\{\delta : (V[R_t(x_t)] - \delta g_{V,t}, E[R_t(x_t)] + \delta g_{E,t}) \in \Re_t\},$$

is called the shortage function at time period t for portfolio x_t in the direction of vector $g_t = (g_{V,t}, g_{E,t})$.

The shortage function looks for improvements in the direction of both an increased mean return and a reduced risk. Notice that the efficiency improving direction vector g_t depends on time. The purpose of this time-dependency is to cater for the potentially changing preferences of the investor over time. The pertinence of the shortage function as a portfolio management efficiency indicator results from its properties. In particular, this indicator characterizes the Markowitz frontier, is weakly monotonic and continuous on $\Im$, and generalizes the Morey and Morey (1999) approaches who look either for return expansions or risk reductions only (see Briec et al. (2004) for details). Notice that if $g_t = 0$, then $S_t(x_t; g_t) = +\infty$. In general, we assume that $g_t \neq 0$.

Markowitz (1952) also proposed an optimization program in a dual, MV utility based framework to determine the portfolio corresponding to a given degree of risk-aversion. To provide a dual interpretation of the shortage function, Briec et al. (2004) also define a MV indirect utility function as the support function of the Markowitz frontier. From the duality result by Luenberger (1995), who connected expenditure and shortage functions, these same authors derive the shortage function from the indirect MV utility function and conversely through a dual pair of relationships. Following this dual relation, it is also possible to disentangle between various efficiency notions when evaluating potential improvements in portfolios. By analogy with other domains in economics, Briec et al. (2004) distinguish formally between (i) portfolio efficiency, (ii) allocative efficiency, and (iii) overall efficiency. For reasons of space and since the empirical application ignores the utility approach, we provide the intuition behind this duality relationship and the ensuing efficiency taxonomy in Appendix 1 or refer the reader to Briec et al. (2004) for details.

3.2. Portfolio performance change in discrete time: A Luenberger portfolio productivity indicator

This subsection is concerned with the dynamic study of portfolio performance in discrete time. Using a recent Luenberger productivity indicator based on some combinations of shortage functions (see Chambers, 2002), our new proposal applies this Luenberger indicator to measuring dynamic portfolio performance.

However, this requires an adaptation of Definition 3.1 of the shortage function to a dynamic context.

Definition 3.2. Given two time periods a and b, the function $S_b : \Im \times \mathbb{R}^2_+ \to \mathbb{R} \cup \{-\infty, +\infty\}$ defined by

$$S_b(x_a; g_a) = \sup_\delta\{\delta : (V[R_a(x_a)] - \delta g_{V,a}, E[R_a(x_a)] + \delta g_{E,a}) \in \Re_b\}, \quad (5)$$

is called the shortage function at time period b in the direction of vector $g_a = (g_{V,a}, g_{E,a})$ for portfolio x_a calculated at time period a.

Remark that $E[R_a(x_a)]$ stands for the expected return of portfolio x_a calculated at time period a, and an analogous interpretation applies to the variance. Notice also that if $t = a = b$, then Definition 3.2 corresponds to Definition 3.1. In this case, the value of δ is always positive. However, for different time periods, this need not be the case. As in Definition 3.1, the direction vector g_a is assumed to be distinct from zero in the general case, although the value of $+\infty$ can be assigned to $S_b(x_a; 0)$. Furthermore, $S_b(x_a, g_a) = -\infty$ if there is no scalar δ such that $(V[R_a(x_a)] - \delta g_{V,a}, E[R_a(x_a)] + \delta g_{E,a}) \in \Re_b$. In the following, we are especially interested in the evolution of the shortage function for two consecutive periods, that is: $(a, b) \in \{t, t+1\} \times \{t, t+1\}$.

The difference derived from expression (5) between two periods at $a = t$ and $a = t + 1$, given a representation set at $b = t$ yields:

$$\Delta_t(x_t, x_{t+1}; g_t, g_{t+1}) = S_t(x_t; g_t) - S_t(x_{t+1}; g_{t+1}). \quad (6)$$

This period t productivity indicator simply computes a difference in the distances between the MV portfolio representations in periods t and $t + 1$ relative to the portfolio frontier in period t. Considering the representation set at $b = t + 1$, we can compute a similar indicator:

$$\Delta_{t+1}(x_t, x_{t+1}; g_t, g_{t+1}) = S_{t+1}(x_t; g_t) - S_{t+1}(x_{t+1}; g_{t+1}). \quad (7)$$

Relative to the portfolio frontier in period $t + 1$, this period $t + 1$ productivity indicator calculates the difference in the distances between the MV portfolio representations in periods t and $t + 1$.

Notice that both these indicators mix various shortage functions which themselves are based on forward and/or backward looking return and other moment information. For example, one can consider $S_{t+1}(x_t; g_t)$ as the ex-ante error made by a portfolio manager in choosing his portfolio weights at time t with respect to information available at time $t + 1$, while $S_{t+1}(x_{t+1}; g_{t+1})$ expresses the counterpart ex-post error observed at time $t + 1$.

To avoid an arbitrary choice between time periods, it is natural (see, e.g., Chambers, 2002) to take the arithmetic mean of the two indicators defined above to obtain the discrete time Luenberger portfolio productivity indicator of performance change

$$L(x_t, x_{t+1}; g_t, g_{t+1}) = \frac{1}{2}[\Delta_t(x_t, x_{t+1}; g_t, g_{t+1}) + \Delta_{t+1}(x_t, x_{t+1}; g_t, g_{t+1})], \quad (8)$$

which is the portfolio analogue of a Luenberger productivity indicator.[11] This portfolio performance change can be equivalently decomposed as:

$$L(x_t, x_{t+1}; g_t, g_{t+1}) = E(x_t, x_{t+1}; g_t, g_{t+1}) + F(x_t, x_{t+1}; g_t, g_{t+1}), \quad (9)$$

with

$$E(x_t, x_{t+1}; g_t, g_{t+1}) = S_t(x_t; g_t) - S_{t+1}(x_{t+1}; g_{t+1}), \quad (10)$$

and

[10] The shortage function has been introduced by Luenberger (1992, 1995) in a production theory context where it measures the distance between some point of the production possibility set and the Pareto frontier.

[11] Notice that the Luenberger productivity indicator does not satisfy circularity in this formulation. There are various ways to make it circular. Furthermore, following Diewert (2005), observe that indexes are based on ratios, while indicators are based on differences. Ratio and difference approaches to index numbers differ in terms of basic properties of practical significance: e.g., (i) ratios are unit invariant, differences are not, (ii) differences are invariant to changes in origin, ratios are not, (iii) ratios have difficulties handling zeros, differences have not, etc. In general, a variety of well-known issues in index theory (see, e.g., Diewert, 2005) can probably shed light on some new problems that may crop up when transposing index numbers into portfolio theory.

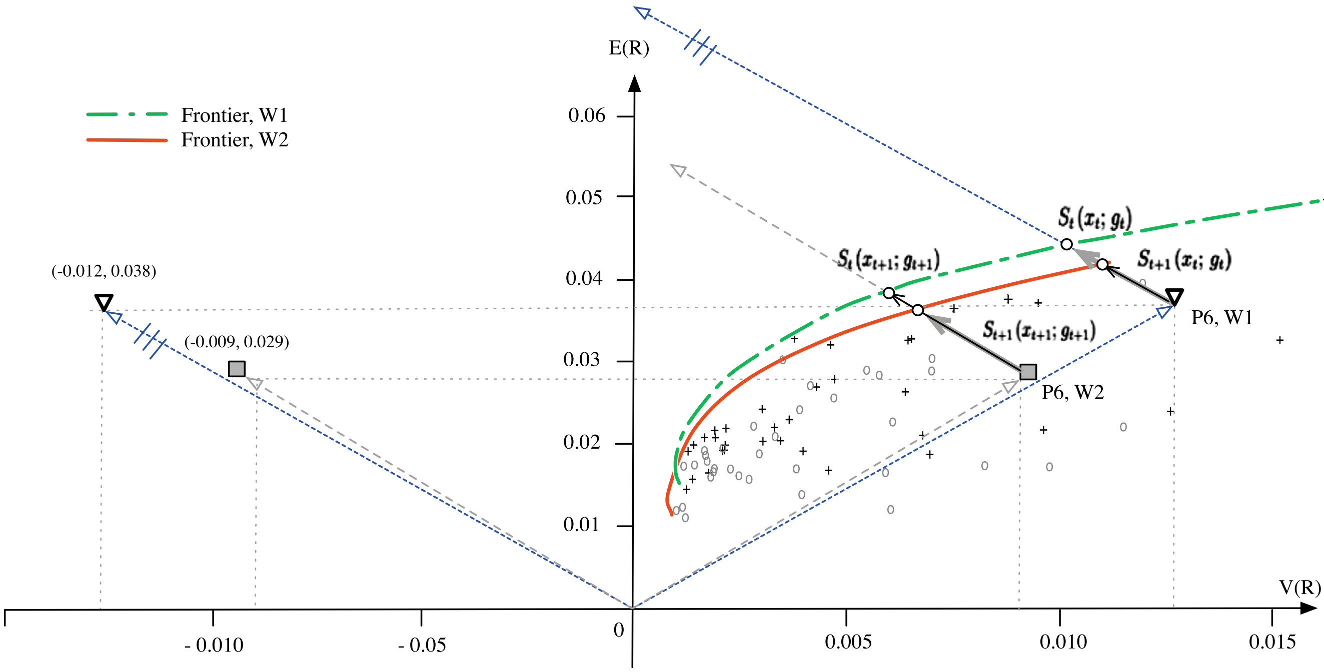

Fig. 2. Luenberger portfolio productivity indicator and its decomposition: Portfolio No. 6.

$$F(x_t, x_{t+1}; g_t, g_{t+1}) = \frac{1}{2}[(S_{t+1}(x_{t+1}; g_{t+1}) - S_t(x_{t+1}; g_{t+1})) + (S_{t+1}(x_t; g_t) - S_t(x_t; g_t))]. \quad (11)$$

In this decomposition, $E(\cdot)$ measures the efficiency change of the shortage functions between periods t and $t+1$, while $F(\cdot)$ captures the average change in portfolio performance between the two periods evaluated at the portfolio composition in $t+1$ and at the portfolio composition in t.[12] Hence, Eq. (9) decomposes portfolio performance change into two components: one representing efficiency change relative to a moving portfolio frontier ($E(\cdot)$), another indicating the average change in the portfolio frontier itself ($F(\cdot)$). This decomposition offers a measurement framework for financial market performance gauging because: on the one hand, $E(\cdot)$ captures the performance of the fund managers over time relative to a shifting portfolio frontier, and on the other hand, $F(\cdot)$ indicates how the financial market itself has locally changed over time and enlarges or reduces the opportunities available to investors. When the Luenberger indicator of portfolio performance change $L(\cdot)$ or any of its components ($E(\cdot)$ or $F(\cdot)$) is positive (negative), then portfolio performance increases (decreases) between the two time periods considered.

Fig. 2 illustrates the above performance indicator for a basic MV portfolio model with $g_s = (V[R_s(x_s)], E[R_s(x_s)])$ for $s = t, t+1$. More precisely, we illustrate the Luenberger indicator and its decomposition with the help of a certain portfolio 6 over two overlapping time windows $W1$ and $W2$.[13] Fig. 2 plots two MV frontiers computed with the returns in the sample over $W1$ and $W2$. Portfolios are plotted using crosses in $W1$ and dots in $W2$, except $P6$ that is once plotted with a black triangle in $W1$ and once with a gray square in $W2$. Arrows indicate the respective distances towards the frontiers in both periods ($S_t(x_t; g_t), S_{t+1}(x_t; g_t), S_t(x_{t+1}; g_{t+1})$, and $S_{t+1}(x_{t+1}; g_{t+1})$ as defined before). The Luenberger indicator must be constructed from its components: $S_t(x_t; g_t) = 0.3795, S_{t+1}(x_{t+1}; g_{t+1}) = 0.3475$, $S_{t+1}(x_t; g_t) = 0.3053$, and $S_t(x_{t+1}; g_{t+1}) = 0.4191$. To obtain $E(\cdot)$ (see Eq. (10)), it suffices to compute: $0.3795 - 0.3475 = 0.0320$. Clearly, this portfolio has moved closer to the portfolio frontier over time yielding a positive $E(\cdot)$. Computing the $F(\cdot)$ (see Eq. (11)) requires the following calculations: $0.5 \times ((0.3475 - 0.4191) + (0.3053 - 0.3795)) = -0.0729$. This negative number simply reflects the productivity decrease due to the inward shift of the portfolio frontier around portfolio 6. Notice that this inward shift of the portfolio frontier is not a global phenomenon: it does not affect the lower risk-return combinations. The Luenberger indicator is simply the sum of these two components: $0.0320 + (-0.0729) = -0.0409$. In this case, the improvement of the $E(\cdot)$ is overruled by the local deterioration of the $F(\cdot)$ and we end up with a negative portfolio frontier productivity change.

Turning to computational matters, the representation set $\Re_t$ (see Eq. (3)) is used to directly compute the various shortage functions and thus the Luenberger indicators by recourse to standard quadratic programming (QP). Assume a sample of m portfolios $x_t^1, x_t^2, \ldots, x_t^m$ is observed over a given finite time horizon $t = 1, \ldots, T$. Now, consider a specific portfolio x_t^k for $k \in \{1, \ldots, m\}$ at time period t whose performance needs gauging. To calculate the Luenberger indicator, the four different shortage functions composing it must be computed by solving a QP for each. To solve for $S_t(x_t; g_t)$ in a basic MV model, the following QP must be computed:

$$\begin{aligned} \max \quad & \delta \qquad (12) \\ \text{s.t.} \quad & E[R_t(x_t)] + \delta g_{E,t} \leqslant \sum_{i=1,\ldots,n} y_{i,t} E[R_{i,t}] \\ & V[R_t(x_t)] - \delta g_{V,t} \geqslant \sum_{i,j} \Omega_{i,j,t} y_{i,t} y_{j,t} \\ & \sum_{i=1,\ldots,n} y_{i,t} = 1, y_{i,t} \geqslant 0, \delta \geqslant 0, \quad i = 1, \ldots, n, \end{aligned}$$

where δ and $y_{i,t}, (i = 1, \ldots, n)$ are decision variables. This QP is then solved for each portfolio with respect to the portfolio set at periods t and $t+1$. For the latter computation, one simply replaces the left-hand side of the first two constraints by the return and risk of the evaluated portfolio in period $t+1$ and also the corresponding direc-

[12] From here onwards, the arguments of the functions defining the Luenberger indicator and its components are suppressed.

[13] This example is drawn from the empirical analysis in Sections 4 and 5. The two time windows range respectively from 1934/01 till 1937/01 ($W1$) and 1934/02 till 1937/02 ($W2$).

tion vector g_{t+1} to end up with $S_t(x_{t+1}; g_{t+1})$. To compute the remaining two shortage functions, one proceeds as follows. To obtain $S_{t+1}(x_{t+1}; g_{t+1})$, all that is needed is to replace the subscript t by $t+1$ everywhere in (12). $S_{t+1}(x_t; g_t)$ is found by replacing the returns, variances and covariances at time period t, occurring on the right-hand side of the first two constraints of model (12), by those computed at time period $t+1$.

We add two remarks on computational issues. First, while in principle several options are available for the choice of direction vector (see Briec et al. (2004) for details), we opt here to employ the observation under evaluation itself, that is, $g_t = (g_{V,t}, g_{E,t}) = (V[R_t(x_t)], |E[R_t(x_t)]|)$.[14] In this case, the shortage function measures the maximum percentage of simultaneous risk reduction and expected return augmentation. Second, it is well-known that in certain cases the shortage function is not well-defined and achieves a value of infinity (e.g., Luenberger, 1995). Focusing on the choice of direction vector, Briec and Kerstens (2009) show that the shortage function, one of the most general distance functions available in the literature so far, may not achieve its distance in the general case where a point need not be part of technology and where the direction vector can take any value. As a consequence, the feasibility of the Luenberger productivity indicator can in general not be guaranteed.[15] Apart from reporting any eventual infeasibilities, these authors show that there is no easy solution in general. Notice that the efficiency measures proposed by Morey and Morey (1999), as special cases of the shortage function approach, are even more vulnerable to the infeasibility issue. Its incidence in a portfolio context has never been reported.

Finally, though the Luenberger indicator is not based on a utility approach, it is important to realize that the performance changes traced over time do reflect gains and losses in utility. This interpretation is developed in Briec et al. (2004).

4. Research methodology: Implementation strategy and data

To illustrate how the Luenberger indicator and its components can serve to track individual fund managers' performance, we opt for a mimicking portfolio approach (Fama and French, 1996). This approach employs portfolios categorized on some variable or combination of variables of interest (e.g., Fama and French (1996) form portfolios on firm size and book-to-market equity, while Fama and French (1997) do the same on industry). In our case, we employ portfolios formed on specific factors or styles. To compose these portfolios and compute the corresponding value-weighted monthly returns, the underlying universe of financial assets is restricted to all stocks listed on the main North American stock markets (in particular, NYSE, AMEX and NASDAQ). In particular, we use a data set made available by French consisting in series of monthly returns from January 1931 to August 2007 for 36 value-weighted (hence, potentially non-optimal) portfolios denoted $P1, P2, \ldots, P36$ and formed on specific factors or styles.[16]

This data set has four important characteristics: (i) the asset universe is common to all portfolios and available over a long time period, (ii) portfolios are not handled by real fund managers over a certain relatively short time span, but represent a variety of management styles that could have been implemented over a long run by some idealized manager, (iii) the value-weighted and non-optimized nature of the portfolios potentially allows for a wide scope of inefficiencies, and (iv) the portfolios have a known time-frame (i.e., a month), since they are recomposed each month or each several months depending on factors or styles. By contrast, real world funds have the disadvantage of having no such natural time unit (e.g., the frequency of rescheduling is (i) hard to infer precisely from mission statements, (ii) can vary slightly over time, and (iii) need not coincide across funds).[17]

To test the capabilities of our new methodology for tracking these inefficiencies, we compute the performance of these idealized funds over a series of sliding time windows with respect to a common fund frontier composed of all selected mimicking portfolios. Since the reallocation of assets within the sample of portfolios is at least partly asynchronous, the resulting heterogeneity in portfolio performance under idealized circumstances forms a perfect level playing field to assess the long run success of certain portfolio management strategies conditioned on styles or factors. In particular, this framework opens up two interesting perspectives in the empirical part that are specific to these methodological choices.

First, we compare these portfolios in terms of the Luenberger indicator and its decomposition over a very long time period and under identical circumstances and contrast it to more traditional performance appraisal tools. Borrowing from the existing literature, we use the Sharpe (*Sharpe*) and Sortino (*Sort*) ratios to evaluate the MV and MVS models, while we opt for the more recent Omega (*Omega*) measure (Kazemi et al., 2004) as a counterpart for the MVSK model.[18] Obviously, a plethora of other traditional financial indices could have been used instead (e.g., the Omega measure could have been replaced by the modified Sharpe ratio proposed by Gregoriou and Gueyie (2003)). We then define their percentage changes ($\Delta_t Sharpe, \Delta_t Sort$ and $\Delta_t Omega$) to have a traditional analogue to the difference-based Luenberger portfolio productivity indicator that also has a percentage interpretation for our choice of direction vector.

Second, the decomposition of the Luenberger indicator provides a unique tool for the long run assessment of the relative success of implementing different portfolio strategies (e.g., based on various styles, factors, etc.). In particular, the efficiency change component ($E(\cdot)$) provides an alternative, but particularly suitable measurement tool to detect the eventual ability of fund managers for stock picking and market timing, since the measurement is not contaminated by the change in the financial market (i.e., it is separated from the frontier change ($F(\cdot)$)).

With a given set of N portfolios, Disatnik and Benninga (2007) underscore the importance to use a minimal size for the time window of $N+1$ to avoid the most dramatic estimation errors in the

[14] Absolute values for return allow for both positive and negative initial data.

[15] This is related to the property of determinateness in index theory which can be loosely stated as requiring that an index remains well-defined when any of its arguments is not.

[16] The following list provides succinct information on how these 36 portfolios have been composed: (1) Fama–French Benchmark (P1–P6): below and above medium size market equity (ME) portfolios based on growth, neutral and value (according to book-to-market (BTM)) portfolios ; (2) size (P7–P11): five portfolios (one per quintile) based on size (ME); (3) growth (P12–P16): five portfolios (one per quintile) based on BTM; (4) dividend yield (P17–P21): five portfolios (one per quintile) based on dividend yield; (5) momentum (P22): picking well-performing stocks from the past; (6) short-term reversal (P23): picking poor-performing stocks from the near past; (7) long term reversal (P24): picking poor-performing stocks from the more distant past; and (8) industry portfolios (P25–P36): portfolios mimicking returns in 12 different industries. More information is available on the web pages of French: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[17] In Appendix 2 we apply the Luenberger productivity indicator to a limited sample of real world funds.

[18] The Omega measure is among the few traditional performance measures that account for all moments of the return distribution (e.g., skewness and kurtosis). It is computed as follows:

$$Omega = \frac{\int_L^b (1 - F(r))dr}{\int_a^L (F(r))dr} \tag{13}$$

where $F(r)$ is the cumulative distribution of returns between a and b, L is the loss threshold: above this threshold returns are considered as gains, below as losses. Thus, Omega can be interpreted as a ratio of gains over losses. The threshold L is here specified as zero.

variance–covariance matrix.[19] Hence, all computations are performed with the same time window of 37 months.[20] The sliding tick for this window is one month. Therefore, since we dispose of 920 months in the data set, we end up with 883 time windows.[21] We also use a 3-month T-Bill as reference for the risk-free rate in the traditional performance measures.[22] These data are obtained from the Federal Reserve Board and are only available since January 1934. Consequently, changes in the traditional ratios ($\Delta_t Sharpe, \Delta_t Sort$ and $\Delta_t Omega$) can only be computed from January 1937 onwards. This difference in availability only affects the comparisons between these traditional measures and the Luenberger portfolio productivity indicator.

Thus, given that all 36 portfolios must be evaluated with 4 different shortage functions over 883 time windows, we end up with 127,152 optimizations in total for the MV model and an equal amount for the MVS and MVSK models. Recall that in the case of the MV (MVS or MVSK) model, each portfolio is projected using a shortage function simultaneously looking for return (and skewness) augmentation and risk (and kurtosis) reduction. Notice the computational advantage of using efficiency measures, since it would be more difficult to compare 883 complete MV frontiers with one another (while ignoring the impossibility to do anything similar in the MVS or MVSK cases). The proposed approach only needs the projections of these 36 portfolios in each of the 883 time windows (the remainder of the MV, MVS or MVSK frontiers can be safely neglected).

Notice furthermore that the incidence of the infeasibility problem mentioned before, turns out to be rather minor: we observe infeasibilities for only 165 (i.e., $0.519\% = 165/(883 \times 36)$), 201 (i.e., $0.632\% = 201/(883 \times 36)$) and 1224 (i.e., $3.850\% = 1224/(883 \times 36)$) portfolios in the MV, MVS, respectively the MVSK models. Thus, the problem is rather small in this data base.

5. Empirical results

This section scrutinises these portfolios in terms of their MV, MVS and MVSK Luenberger portfolio productivity indicators, and also compares these to the $\Delta_t Sharpe$, $\Delta_t Sort$, respectively $\Delta_t Omega$ indicators.[23]

A first part of the analysis searches for a common ground in the information provided by this Luenberger productivity indicator and its counterpart traditional performance measures. The idea is to identify whether or not these two categories of performance gauges offer similar results. Rank correlations are computed over the period 02/1937 to 08/2007 (for data availability reasons) between on the one hand $L(\cdot)$ and on the other hand the $\Delta_t Sharpe$ indicator (MV), the $\Delta_t Sort$ indicator (MVS) and the $\Delta_t Omega$ indicator (MVSK). To impose minimal assumptions, these correlations are evaluated by a Spearman rho test. Results are presented in Table 1.

Looking at Table 1, one can draw two conclusions. First, the Luenberger productivity indicator is rather highly positively correlated with all three traditional indicators. This shows that our new approach is not disconnected from existing performance indicators. Notice that while the correlation with the $\Delta_t Sharpe$ indicator (MV) is higher than with the $\Delta_t Sort$ indicator (MVS), the highest correlation is in fact found with the $\Delta_t Omega$ indicator. This seems to point out that the Luenberger indicator captures essential features of the whole return distribution. Second, with regard to its components, frontier change is more strongly positively associated with the traditional performance indicators than the technical efficiency change component. Thus, traditional indicators seem to capture some changes in portfolio frontiers, but have a rather hard time to adequately assess the individual contributions of fund management itself.[24]

Keeping in mind that traditional measures are unable to distinguish the contribution of portfolio managers to the performance evolution, while the Luenberger portfolio productivity indicator and its decomposition allow for such a distinction, we now try to test the relevance of this decomposition. Two questions are considered: (i) is the evolution of $L(\cdot), E(\cdot)$ and $F(\cdot)$ due to mere chance, and (ii) do the series of $L(\cdot), E(\cdot)$ and $F(\cdot)$ have a mean that is different from zero? While the first question is concerned with the detection of any significant influence of portfolio managers on the Luenberger and its components, the second question focuses on the size of any eventual effect.

One basic idea here is simply to identify, if possible, some styles that perform well in terms of efficiency over time (in line with a research stream pioneered by McDonald (1974)). Moreover, since all of these mimicking portfolios belong to a more general active management style (these portfolios being rebalanced on some regular basis), our results could shed some light on the controversy regarding the utility/vacuity of active management. While it is frequently reported that actively managed portfolios fail to outperform passive counterpart strategies (see, e.g., Gruber, 1996), some researchers do find some value added for active mutual fund management (e.g., Wermers, 2000). Thus, while we do not expect reporting portfolios with significant non-zero $L(\cdot)$ (given efficient markets), we wonder whether some styles could exhibit some non-zero $E(\cdot)$. Obviously, positive improvements in $E(\cdot)$ could indicate expertise among some portfolio managers (at least over short periods of time) to push portfolios towards the moving portfolio frontier target, while a negative result could point to their inability to do so.

To answer the first question, we utilize a Wald-Wolfowitz run test. Results are proposed in Table 2. Notice that in the remainder, we only report significant results. Looking at the decomposition, a first major result is that most portfolios exhibit non-random $E(\cdot)$ series in all three models. By contrast, $F(\cdot)$ appears to be almost completely random, as could be expected from efficient market theory. Second, the Luenberger indicator $L(\cdot)$, as the sum of both above components, is mainly non-random for the two first portfolio families. These are in particular, the Fama–French Benchmark portfolios 3 (neither in MV, nor in MVSK), 4, 5 and 6 (i.e., mainly those that are above the median size, whatever their position in terms of BTM) and portfolios 7, 8, 9 and 10 (neither in MVS, nor in MVSK) (i.e., P7–P9 are portfolios composed within the subset of the 60% smallest firms).

The second question is answered using a Wilcoxon test for differences. Over the whole time period, we cannot report any portfolio that has non-zero performance indicators except P31 (a significant $L(\cdot)$ in MV) and P23 (a significant $E(\cdot)$ in MVS and in MVSK). Of course, this is in line with the efficient market hypothesis as well, since it is hard to imagine that the portfolio mimicking approach could generate and sustain superior results over such a

[19] This minimum requirement ensures, for instance, that the covariance matrix can be inverted. If this condition is not respected, estimation errors could bias results (see, e.g., Michaud, 1989).

[20] Robust methods for the estimation of the variance–covariance matrix can, for instance, be found in DeMiguel and Nogales (2009). To the best our knowledge, however, the transposition to higher co-moment tensor estimations is not yet available.

[21] The first time window ranges over the interval [01/1931, 01/1934] and the last one over the interval [08/2004, 08/2007].

[22] Furthermore, these risk-free rates are annualized and converted to a monthly basis.

[23] Appendix 2 discusses the empirical results pertaining to the real world funds and comments upon some contrasting results with the mimicking portfolio approach.

[24] Of course, this aggregate result is accompanied by quite some variations among portfolios. For instance, portfolios 21, 24, and 3 Industry portfolios (28, 31 and 32) even exhibit a non-significant and close to zero rank correlation with regard to the technical efficiency change component, even though all of these are significantly correlated with the frontier change.

Table 1
Portfolios with significant correlations between $L(x_t, x_{t+1}, g_t, g_{t+1})$ and ΔSharpe (MV), ΔSortino (MVS) resp. ΔOmega (MVSK).

Portfolio group	Port. No.	Δ_t*Sharpe* (MV)			Δ_t*Sort* (MVS)			Δ_t*Omega* (MVSK)		
		$E(.)$	$F(.)$	$L(.)$	$E(.)$	$F(.)$	$L(.)$	$E(.)$	$F(.)$	$L(.)$
FF. Benchmark	1	0.3996	0.4366	0.5588	0.3301	0.3543	0.4490	0.3422	0.5704	0.6109
	2	0.2158	0.3737	0.4244	0.3291	0.4130	0.5025	0.3518	0.6231	0.6920
	3	0.1264	0.4060	0.3887	0.3358	0.5099	0.6079	0.3357	0.6991	0.7520
	4	0.2506	0.4373	0.4193	0.4012	0.3711	0.4920	0.4053	0.5722	0.6097
	5	0.1505	0.4018	0.3831	0.4092	0.4908	0.5873	0.4369	0.7056	0.7576
	6	0.1230	0.3665	0.3516	0.3432	0.4952	0.5931	0.3389	0.7049	0.7756
Size	7	0.1266	0.3528	0.3236	0.2850	0.4049	0.4896	0.2666	0.5131	0.5897
	8	0.1963	0.4302	0.4190	0.4735	0.4830	0.6194	0.4646	0.7083	0.7657
	9	0.2583	0.4602	0.4628	0.4413	0.5092	0.6044	0.4633	0.7330	0.7696
	10	0.2999	0.4489	0.4789	0.4585	0.5693	0.6478	0.5139	0.7613	0.8084
	11	0.3141	0.3980	0.4865	0.2831	0.3165	0.3968	0.3060	0.5553	0.5947
Growth	12	0.3929	0.4404	0.5531	0.3576	0.3471	0.4526	0.3786	0.5878	0.6349
	13	0.3179	0.3969	0.4689	0.3741	0.4643	0.5600	0.3547	0.6248	0.6534
	14	0.1520	0.4089	0.4244	0.3060	0.4795	0.5694	0.3501	0.6630	0.7471
	15	0.1527	0.3855	0.3911	0.3667	0.5307	0.6159	0.3926	0.7572	0.8279
	16	0.1281	0.4223	0.3850	0.3375	0.4436	0.5505	0.3524	0.6680	0.7187
Dividend Yield	17	0.3951	0.4664	0.5597	0.4461	0.4494	0.5766	0.4245	0.6510	0.7041
	18	0.3104	0.4212	0.4864	0.3159	0.3770	0.4683	0.4049	0.5866	0.6636
	19	0.2226	0.3740	0.4490	0.2115	0.4047	0.4660	0.2940	0.6208	0.6730
	20	0.1364	0.3403	0.3952	0.2371	0.4030	0.5170	0.2528	0.6192	0.6869
	21	0.0475 *(0.1672)*	0.3759	0.3630	0.1752	0.3854	0.4184	0.2272	0.5421	0.5968
Momentum	22	0.1989	0.3847	0.4118	0.4173	0.5125	0.6303	0.5023	0.7255	0.8536
ST Reversal	23	0.2052	0.4333	0.4196	0.3666	0.4154	0.5269	0.3567	0.6102	0.7057
LT Reversal	24	0.0354 *(0.3095)*	0.3400	0.2721	0.1548	0.2790	0.3384	0.2037	0.4452	0.5118
Industry	25	0.0998	0.3418	0.3464	0.2364	0.3871	0.4383	0.2374	0.5250	0.5822
	26	0.1754	0.4289	0.4043	0.2502	0.2004	0.2942	0.2314	0.2443	0.3234
	27	0.3038	0.4832	0.5088	0.3292	0.3963	0.4678	0.4287	0.6557	0.7133
	28	−0.0412 *(0.2352)*	0.3145	0.2199	0.2042	0.2500	0.3279	0.1840	0.4055	0.4751
	29	0.2621	0.3974	0.4640	0.2210	0.2843	0.3709	0.2510	0.4521	0.5300
	30	0.2653	0.4363	0.4839	0.2049	0.2211	0.2850	0.2031	0.3146	0.3407
	31	0.0065 *(0.8508)*	0.3288	0.3015	0.1284	0.1865	0.2698	0.1516	0.2473	0.3299
	32	−0.0528 *(0.1250)*	0.3036	0.2627	0.1421	0.1466	0.2016	0.2112	0.2957	0.3878
	33	0.2172	0.4347	0.4609	0.3245	0.4045	0.5002	0.3723	0.5451	0.6430
	34	0.1079	0.4033	0.4057	0.1538	0.2589	0.3541	0.2015	0.3969	0.4626
	35	0.1527	0.4158	0.4055	0.3486	0.4006	0.5206	0.3610	0.5875	0.6583
	36	0.2695	0.4083	0.4415	0.4034	0.3379	0.4729	0.3247	0.4704	0.5258
Overall		0.1920	0.4001	0.4158	0.1414	0.3154	0.3233	0.3232	0.5491	0.6122

Note: Spearman Correlation coefficient with H_0: $\rho = 0$. Absence of *p*-value indicates a significance at 1% threshold.
To save space, we denote $E(x_t, x_{t+1}; g_t, g_{t+1})$, $F(x_t, x_{t+1}; g_t, g_{t+1})$, and $L(x_t, x_{t+1}; g_t, g_{t+1})$ by $E(.)$, $F(.)$, respectively $L(.)$. This is done in all tables.

long run. However, in a sufficiently short time horizon (1–3 years: see, e.g., Brown and Goetzmann, 1995) and sometimes over longer periods (5–10 years: e.g., Elton et al., 1996), one can imagine that some portfolios (e.g., styles, etc.) may perform well because, for a variety of reasons, their profile fits into some market niche favored by the economy. Therefore, we look at the short run by fixing a period consisting of the last ten years. The Wilcoxon test is recomputed and results are reported in Table 3.

While no portfolio gets a significant $E(\cdot)$ in MV, and only one (P35) in MVS and in MVSK, quite a few obtain non-zero $L(\cdot)$ and $F(\cdot)$. Notice that not a single portfolio obtains a non-zero Δ_t*Sharpe* or Δ_t*Sort* indicator over the same time span. These portfolios obtain a significant Luenberger indicator value, not because of any capability from the idealized manager, but simply due to changes in the market that temporarily and locally favor certain niches in the portfolio set. Combining this information with the result regarding the first question, one can conjecture that the non-random $E(\cdot)$ found there must be caused by some coincidentally favorable circumstances situated in some sub-period(s) different from the last ten years. Among these results, one also notices that there is no evidence supporting the relative interest to invest in high book-to-market portfolios (i.e., value portfolios, P15 and P16). This result contrasts with Lakonishok et al. (1994) who provide contradictory illustrations. One explanation for this difference could be the mechanical behavior of our virtual managers who consistently follow certain management styles. This style consistency is known to be insufficient to achieve good performance levels (see Asness et al., 2000): for instance, some appropriately timed rotation between growth and value styles seems necessary to obtain such good results.

Finally, knowing that non-zero performance is at best only observable in the short-term, we wonder whether there is any time-dependency within these indicator-based performance results within the same 10 year time span. This question relates to the more general issue of performance persistence in portfolio management. An enormous literature has been devoted to this subject ever since Jensen (1968) illustrated the virtual impossibility to outperform the market over long periods and on a regular basis. We test for possible persistence in performance for short periods of time using first-order autocorrelation regressions for efficiency change, frontier change and the Luenberger indicator for MV, MVS and MVSK models. We basically find that non-zero performance in these non-optimized mimicking portfolios cannot be sustained over time (see Appendix 3 for details).

6. Conclusions

The main objective of this contribution is to introduce a general method for measuring the evolution of portfolio efficiency over

Table 2
Run tests for the Luenberger indicator and its components (whole sample).

Portfolio group	Port. No.	Mean-variance						Mean-variance-skewness						Mean-variance-skewness-kurtosis					
		$E(.)$		$F(.)$		$L(.)$		$E(.)$		$F(.)$		$L(.)$		$E(.)$		$F(.)$		$L(.)$	
		Z	p-value	Z	p-value	Z	p-value	Z	p-value	Z	p-value	Z	p-value	Z	p-value	Z	p-value	Z	p-value
FF. Benchmarks	3	–	–	–	–	–	–	−1.115	0.2648	−1.170	0.2419	−1.716	0.0862*	–	–	–	–	–	–
	4	−3.000	0.0027***	−1.448	0.1477	−3.337	0.0008***	−2.180	0.0292**	−0.820	0.4117	−3.468	0.0005***	−1.658	0.0974*	−0.332	0.7402	−2.971	0.0030***
	5	−2.789	0.0053***	−1.116	0.2645	−4.634	0.0000***	−1.649	0.0991*	−1.550	0.1212	−5.031	0.0000***	−2.081	0.0374*	−0.858	0.3907	−4.671	0.0000***
	6	−3.764	0.0002***	−1.689	0.0912*	−3.045	0.0023***	−4.476	0.0000***	−1.680	0.0929***	−2.690	0.0071***	−5.289	0.0000***	−1.819	0.0689*	−2.927	0.0034***
Size	7	−6.503	0.0000***	−2.063	0.0391*	−4.354	0.0000***	−8.303	0.0000***	−2.294	0.0218*	−3.915	0.0001***	−8.081	0.0000***	−2.161	0.0307*	−3.511	0.0004***
	8	−2.668	0.0076***	−1.284	0.1990	−3.169	0.0015***	−1.811	0.0701*	−1.274	0.2029	−4.067	0.0000***	−1.627	0.1038	−0.393	0.6941	−3.780	0.0002***
	9	−1.517	0.1293	−1.034	0.3010	−2.655	0.0079***	−0.772	0.4399	−1.314	0.1887	−2.798	0.0051***	−0.807	0.4196	−0.405	0.6858	−2.287	0.0222*
	10	−1.716	0.0861*	−0.027	0.9783	−2.727	0.0064***	–	–	–	–	–	–	–	–	–	–	–	–
Growth	14	–	–	–	–	–	–	−1.986	0.0470*	−0.096	0.9232	−0.096	0.9232	–	–	–	–	–	–
	15	−2.140	0.0323*	0.372	0.7097	1.314	0.1891	−2.188	0.0287*	1.352	0.1765	1.238	0.2159	−2.850	0.0044***	0.434	0.6640	0.595	0.5518
	16	−2.059	0.0395*	0.204	0.8388	−0.966	0.3341	−4.860	0.0000***	−0.776	0.4377	−1.816	0.0693*	−4.861	0.0000***	−0.935	0.3498	−3.094	0.0020***
Dividend yield	17	−1.137	0.2554	0.086	0.9314	−2.221	0.0263*	−1.964	0.0495*	−0.120	0.9048	−1.057	0.2906	−2.133	0.0329*	−0.113	0.9100	−1.341	0.1798
	20	−2.256	0.0241*	−0.676	0.4992	−1.515	0.1298	−2.724	0.0064***	1.225	0.2207	−1.107	0.2681	−2.035	0.0419*	0.871	0.3840	−0.918	0.3588
	21	–	–	–	–	–	–	–	–	–	–	–	–	−1.973	0.0485*	−0.611	0.5410	0.041	0.9674
Momentum	22	−4.495	0.0000***	−0.961	0.3366	−0.919	0.3582	−6.263	0.0000***	−0.805	0.4206	−0.732	0.4642	−6.045	0.0000***	−0.741	0.4588	−0.940	0.3471
ST Reversal	23	−2.473	0.0134*	−1.700	0.0890*	−0.769	0.4421	−6.686	0.0000***	−1.204	0.2288	−0.422	0.6730	−6.380	0.0000***	−1.339	0.1806	−0.557	0.5775
LT Reversal	24	−2.230	0.0258*	−0.683	0.4949	−1.747	0.0806*	−7.022	0.0000***	1.223	0.2212	−0.716	0.4742	−7.280	0.0000***	1.279	0.2008	−0.746	0.4560
Industry	25	−2.765	0.0057***	−1.421	0.1554	−0.745	0.4562	−2.237	0.0253*	0.738	0.4603	0.774	0.4389	−1.661	0.0966*	1.119	0.2631	0.625	0.5318
	26	−2.199	0.0278*	−0.023	0.9820	0.104	0.9174	−6.850	0.0000***	−1.655	0.0979*	−1.643	0.1003	−7.183	0.0000***	−1.418	0.1562	−2.078	0.0377*
	28	−3.410	0.0006***	−0.467	0.6402	0.273	0.7849	−8.426	0.0000***	−0.977	0.3287	0.220	0.8256	−7.794	0.0000***	−0.438	0.6615	1.334	0.1821
	29	−1.683	0.0924*	0.371	0.7108	1.449	0.1474	−1.869	0.0617*	0.453	0.6503	1.387	0.1654	−2.497	0.0125*	0.323	0.7468	1.665	0.0959*
	30	−2.800	0.0051***	−0.334	0.7388	−0.595	0.5519	−5.298	0.0000***	0.409	0.6824	0.304	0.7608	−5.025	0.0000***	1.504	0.1327	0.680	0.4965
	31	−3.740	0.0002***	−0.354	0.7235	−0.564	0.5727	−7.750	0.0000***	−0.554	0.5793	−0.975	0.3294	−7.352	0.0000***	−1.215	0.2243	−1.219	0.2229
	32	–	–	–	–	–	–	−5.008	0.0000***	−0.235	0.8139	−0.492	0.6230	−4.894	0.0000***	−0.505	0.6132	−0.769	0.4420
	33	−2.978	0.0029***	−1.416	0.1568	−0.576	0.5649	−3.486	0.0005***	−0.337	0.7364	−0.601	0.5478	−3.103	0.0019***	−0.846	0.3978	0.102	0.9184
	34	−3.444	0.0006***	0.081	0.9354	−1.307	0.1912	−6.388	0.0000***	0.728	0.4665	0.524	0.6007	−6.156	0.0000***	0.265	0.7909	0.481	0.6305
	35	–	–	–	–	–	–	−1.716	0.0861*	1.498	0.1342	−0.675	0.4998	−1.762	0.0780*	0.867	0.3861	−0.366	0.7146
	36	−1.383	0.1667	−1.651	0.0988*	−1.773	0.0763*	–	–	–	–	–	–	–	–	–	–	–	–

Note: Wald-Wolfowitz run test with H_0: series X follows a random process.
* 10% Signs represent thresholds.
** 5% Signs represent thresholds.
*** 1% Signs represent thresholds.

Table 3
Wilcoxon tests for the Luenberger indicator and its components (last ten years).

Portfolio group	Port. No.	$E(.)$		$F(.)$		$L(.)$	
		W	p-value	W	p-value	W	p-value
Mean-variance							
FF. Benchmark	1	–	–	–	–	0.0028	0.0643*
	2	–	–	0.0058	0.0824*	–	–
Size	11	–	–	0.0047	0.0967*	0.0059	0.0422**
Growth	12	–	–	–	–	0.0006	0.0994*
Dividend yield	19	–	–	0.0075	0.0311**	0.0133	0.0367**
	20	–	–	0.0091	0.0148**	–	–
Industry	25	–	–	0.0077	0.0203**	0.0123	0.0607*
	29	–	–	0.0108	0.0054***	0.0145	0.0178**
	31	–	–	–	–	−0.0002	0.0521*
	36	–	–	0.0065	0.051*	–	–
Mean-variance-skewness							
FF. Benchmark	1	–	–	0.0108	0.0967*	0.0119	0.065*
	19	–	–	0.0101	0.0698*	0.0164	0.0685*
	25	–	–	0.0277	0.0517*	–	–
	28	–	–	−0.0192	0.0580*	−0.0186	0.0889*
	29	–	–	0.0534	0.0841*	0.0571	0.0961*
	34	–	–	–	–	0.0161	0.0967*
	35	0.0023	0.0736*	–	–	–	–
Mean-variance-skewness-kurtosis							
FF. Benchmark	1	–	–	0.01213	0.0967*	0.0133	0.0636*
Dividend yield	19	–	–	0.0135	0.0698*	0.0199	0.0685*
Industry	28	–	–	−0.0219	0.0408**	−0.0219	0.0605*
	34	–	–	–	–	0.0163	0.0931*
	35	0.0025	0.0805*	–	–	–	–

Note: Wilcoxon test with H_0: value is not different of 0.
* 10% Represent thresholds.
** 5% Represent thresholds.
*** 1% Represent thresholds.

time inspired by developments in index theory. Benchmarking portfolios by simultaneously looking for risk (and kurtosis) contraction and mean return (and skewness) augmentation in the MV (MVS and MVSK) model using the shortage function framework, we define a new Luenberger discrete time portfolio productivity indicator. The cardinal virtues of this approach are: (i) it does not require the complete estimation of the efficient frontier and tracing its evolution over time, but simply projects the portfolios on the relevant part of the frontier with the shortage function using non-parametric envelopment methods to obtain an easily interpretable efficiency measure and an ensuing productivity indicator; (ii) the decomposition of the Luenberger portfolio productivity indicator distinguishes between efficiency change and portfolio frontier change. While the latter component measures the local changes in the frontier movements induced by market volatility, the former can in principle capture efficiency changes attributable to the investor or portfolio manager. This efficiency change component allows testing in an alternative, but conceptually promising way the eventual ability of fund managers to generate superior performances, since this measurement is not contaminated by any changes in the financial market itself.

An empirical application on a limited sample of idealized portfolios illustrates the computational feasibility of this general framework in the MV, MVS and MVSK frameworks. Given the mimicking portfolio approach adopted, and the long time period available, we are able to shed some light on the question of the relative performance of implementing different portfolio strategies (e.g., based on various styles, factors, etc.). Summarizing some key empirical results, the Luenberger portfolio productivity indicator is positively correlated with its counterpart traditional performance measures in all three portfolio frameworks. This correlation is probably mainly due to the capacity of traditional measures to track portfolio frontier changes. The latter measures cannot identify the individual contribution of fund management, as it is captured by our technical efficiency change component. Furthermore, most portfolios exhibit non-random efficiency change series in all three portfolio models, while frontier change series are almost completely random. Additionally, the efficiency change series does almost never yield a non-zero performance. By contrast, the frontier change component of some portfolios can be significantly different from zero in the short run, because the market coincidentally seems to create favorable circumstances. Overall, these results are perfectly concordant with efficient market theory and are probably driven by the mimicking portfolio approach which relies in the selected data base on non-optimized rules. Nevertheless, this new framework opens up possibilities to systematically attribute performance and quantify any eventual individual fund manager performance.

Obviously, the current work has some limitations. One restriction is that it does not account for transaction costs, but assumes that portfolios can be reshuffled in every time period to remain in track with the evolving portfolio frontiers. This can in principle be overcome at the cost of complexifying the analysis slightly. However, we do not anticipate any fundamental problem in extending the proposed Luenberger indicator, since all extensions of basic portfolio models could in principle be fitted into the basic shortage function models. Another restriction is that it ignores the revived interest in downside risk (see, e.g., Morton et al. (2006)) or Chen and Wang (2008)). It is likely possible to formulate similar portfolio productivity gauges using partial moments.

Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.jbankfin.2009.12.015.

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