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The Luenberger productivity indicator: An economic specification leading to infeasibilities ☆

Walter Briec [a], Kristiaan Kerstens [b,*]

[a] University of Perpignan, LAMPS, 52 Avenue Villeneuve, F-66860 Perpignan Cedex, France
[b] CNRS-LEM (UMR 8179), IESEG School of Management, 3 rue de la Digue, F-59000 Lille, France

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ABSTRACT

This contribution points out an error in the specification of technology when computing the Luenberger productivity indicator that has been hitherto ignored in the literature. The solution of this problem increases the likelihood that the directional distance functions underlying this productivity indicator are ill-defined.

1. Introduction

Total factor productivity (*TFP*) growth measures traditionally the shifts in technology in a residual way, namely in terms of output growth which remains unexplained by the input growth (Hulten, 2001). Nishimizu and Page (1982) innovated by decomposing *TFP* growth into technical change and technical efficiency change using parametric production frontiers. They realised that ignoring inefficiency may bias *TFP* measurement. Discrete time Malmquist input- and output-oriented productivity indexes based upon Shephardian distance functions (see Caves, Christensen and Diewert, 1982) have been made empirically tractable by Färe et al. (1995). By exploiting the relation between distance functions and radial efficiency measures, these authors suggest computing distance functions using deterministic, non-parametric technologies (as inner bound approximations of the true but unknown technology). Furthermore, these same authors integrate the two-part decomposition of *TFP* of Nishimizu and Page (1982) into this Malmquist productivity index. Meanwhile, dozens of articles have employed this Malmquist productivity index to study productivity change in a wide variety of empirical contexts.

Meanwhile, more general primal productivity indicators have been proposed.[1] Indeed, in a series of articles Chambers et al. (1996), Chambers and Pope (1996) and Chambers (2002) define a Luenberger productivity indicator as a difference-based index of directional distance functions. The latter functions generalize Shephardian distance functions by accounting for both input reductions and output augmentations and they are dual to the profit function. It is possible to define input- and output-oriented versions of this Luenberger indicator as special cases. These indicators can then be interpreted as difference-based versions of their similarly oriented Malmquist productivity indices. Though it is not yet as popular as the Malmquist productivity index, the Luenberger productivity indicator has recently been used as a tool for empirical analysis in a series of articles (e.g., Barros and Peypoch, 2007; Boussemart et al., 2003; Guironnet and Peypoch, 2007; Managi, 2003; Nakano and Managi, 2008, among others).

This contribution points out a basic problem in the computation of the Luenberger productivity indicator that has been hitherto ignored in the existing literature. The solution of this problem increases the probability that the directional distance functions underlying this productivity indicator are ill-defined. The next section defines the basics to formulate the Luenberger productivity indicator, points out the basic problem in its computation, and indicates a way out.

2. Luenberger productivity indicator

Production technology transforms inputs $x=(x_1,\cdots,x_n)\in\mathbb{R}^n_+$ into outputs $y=(y_1,\cdots,y_p)\in\mathbb{R}^p_+$. For each time period t, the production possibility set T summarizes the set of all feasible input and output vectors. This technology can be defined as follows:

$$T^t=\left\{\left(x^t,y^t\right)\in\mathbb{R}^{n+p}_+;\ x^t \text{ can produce } y^t \text{ in period } t\right\}. \tag{2.1}$$

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* Corresponding author. Tel.: +33 320545892; fax: +33 320574855.
E-mail addresses: briec@univ-perp.fr (W. Briec), k.kerstens@ieseg.fr (K. Kerstens).
[1] "Indicators" denote productivity measures based on differences, while "indexes" indicate productivity measures defined as ratios (see Diewert, 2005 for an overview).

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Throughout this note, technology satisfies the following standard assumptions: (T.1) $(0, 0)\in T$, $(0, y)\in T\Rightarrow y=0$ i.e., no free lunch; (T.2) the set $A(x)=\{(u,y)\in T;\ u\leqslant x\}$ of dominating observations is bounded $\forall x\in\mathbb{R}^n_+$, i.e., infinite outputs are not allowed with a finite input vector; (T.3) T is closed; (T.4) $\forall(x,y)\in T$, $(u,v)\geq 0$ and $(x,-y)\leqslant(u,-v)\Rightarrow(u,v)\in T$, i.e., fewer outputs can always be produced with more inputs, and inversely (strong disposal of inputs and outputs); and (T.5) T is convex. Notice that to simplify notation, technology has no time superscript.

One way to characterize technology is the use of distance functions. In an effort to simplify notation, we denote $z=(x,y)\in T$ and $g=(h,k)\in(-\mathbb{R}^n_+)\times\mathbb{R}^p_+$ and which is partitioned in an input and an output direction vector h respectively k. The directional distance function involving a simultaneous input and output variation in the direction of a pre-assigned vector g is defined as:[2]

Definition 1. The function D_T: $\mathbb{R}^{n+p}_+\times(-\mathbb{R}^n_+)\times\mathbb{R}^p_+\rightarrow\mathbb{R}\cup\{-\infty\}\cup\{+\infty\}$ defined by

$$D(z;g)=\begin{cases}\sup\limits_{\delta}\{\delta\in\mathbb{R}:z+\delta g\in T\} & \text{if } z+\delta g\in T \text{ for some } \delta\in\mathbb{R}\\ -\infty & \text{otherwise}\end{cases}$$

is called the directional distance function in the direction of $g=(h,k)$.

Notice that distance functions are related to efficiency measures in that they measure deviations from the boundary of technology. Notice furthermore that, following a tradition in defining this distance function (e.g., Chambers, 2002), we distinguish between the standard case where the distance is achieved and the case where there is no way to achieve the distance. This function has been proven to be a useful tool in applied production analysis. For instance, it allows Chavas and Kim (2007) to shed new light on economies of scope from a primal viewpoint. Furthermore, it provides the defining components of the Luenberger productivity indicator to which we now turn.

To introduce the Luenberger productivity indicator, we now introduce a time superscript into the directional distance function. Let $(a,b)\in\{t,t+1\}\times\{t,t+1\}$, we denote:

$$D_b(z^a;g^a)=\sup_{\delta}\left\{\delta\in\mathbb{R}:z^a+\delta g^a\in T^b\right\}. \tag{2.2}$$

The Luenberger productivity indicator $L(z^t, z^{t+1})$, initially proposed in Chambers et al. (1996), Chambers and Pope (1996) and Chambers (2002), can now be defined as:

$$L\left(z^t,z^{t+1};g^t,g^{t+1}\right)=\frac{1}{2}\Big(D_t\left(z^t;g^t\right)-D_t\left(z^{t+1};g^{t+1}\right)+D_{t+1}\left(z^t;g^t\right)-D_{t+1}\left(z^{t+1};g^{t+1}\right)\Big). \tag{2.3}$$

An arithmetic mean of a Luenberger productivity indicator in base year t and $t+1$ is taken to average out the effect of selecting an arbitrary base year. Productivity growth (decline) is indicated by positive (negative) values. Chambers et al. (1996) also indicate that the Luenberger indicator can be decomposed as follows:

$$\begin{aligned}L\left(z^t,z^{t+1};g^t,g^{t+1}\right)=&\left[D_t\left(z^t;g^t\right)-D_{t+1}\left(z^{t+1};g^{t+1}\right)\right]\\&+\frac{1}{2}\Big[\Big(D_{t+1}\left(z^t;g^t\right)-D_t\left(z^t;g^t\right)\Big)\\&+\Big(D_{t+1}\left(z^{t+1};g^{t+1}\right)-D_t\left(z^{t+1};g^{t+1}\right)\Big)\Big].\end{aligned} \tag{2.4}$$

The expression in the first brackets represents the technical efficiency change (EC), while the terms in the second brackets represent the technological change (TC).

Recently, Chambers et al. (1996) provide programs to compute the Luenberger productivity indicator (see below) using deterministic, non-parametric technologies (see Varian, 1984 and Banker and Maindiratta, 1988). Notice that while it is true that the vast majority of empirical Luenberger productivity studies employ these technologies (e.g., Boussemart et al., 2003 or Guironnet and Peypoch, 2007), this analysis carries immediately over to parametric specifications of technology (see, e.g., Briec and Kerstens, in press). A study based on parametric technology specifications is Fuentes et al. (2001). An example of an empirical productivity study using both non-parametric and parametric technologies is Atkinson et al. (2003).

Let $\mathcal{J}=\{1,\cdots,m\}$ be an index of observations and consider the set of activities $A=\{z^j: j\in\mathcal{J}\}$. Suppose that $(0, 0)\in A$ and $x^j=0\Rightarrow y^j=0$ in order to obey axioms T.1–T.5. The non-parametric estimate $\hat{T}$ of the unknown technology from the observed set of data A is:

$$\hat{T}=\left\{z\in\mathbb{R}^{n+p}_+ : \forall(w,p)\in\mathbb{R}^{n+p}_+,\exists j\in\mathcal{J}\text{ with p}\cdot\text{y}-\text{w}\cdot\text{xVp}\cdot\text{yj}-\text{w}\cdot\text{xj}\right\}. \tag{2.5}$$

This can equivalently be rewritten as:

$$\hat{T}=\left\{z\in\mathbb{R}^{n+p}_+ : \forall(w,p)\in\mathbb{R}^{n+p}_+,\ r\cdot y-w\cdot x\leq\max_{j\in\mathcal{J}}\{p\cdot yj-w\cdot xj\}\right\}. \tag{2.6}$$

From Varian (1984) and Banker and Maindiratta (1988), the primal formulation of this non-parametric technology can be written:

$$\hat{T}=\left\{z\in\mathbb{R}^{n+p}_+ : x\geq X\theta, y\leq Y\theta, 1^m\cdot\theta=1,\theta\geq 0\right\}, \tag{2.7}$$

where X is a $n\times m$ input matrix whose j-th row is x^j; Y is a $p\times m$ output matrix whose j-th row is y^j; and 1^m is the m-dimensional unit vector. The following program computes the directional distance function with respect to technology $\hat{T}$:

$$\overline{D}(z;g)=\sup\{\delta\in\mathbb{R}:x+\delta h\geq X\theta, y+\delta k\leq Y\theta, 1^m.\theta=1,\theta\geq 0\}. \tag{2.8}$$

Notice that to impose constant returns to scale (as proposed in Chambers et al., 1996), it suffices to drop the weight constraint $(1^m\cdot\theta=1)$ on the activity vector (θ). However, whether one assumes constant returns to scale or not, the above program may well not calculate the directional distance function correctly if traditional economic definitions of non-negative outputs must be respected. In fact, it is easy to see that $\overline{D}(z;g)=\sup\{\delta\in\mathbb{R}: z+\delta g\in\hat{T}+K\}$, where $K=\mathbb{R}^n_+\times(-\mathbb{R}^p_+)$. Hence, the constraint $z+\delta g\geq 0$ is missing. A similar approach has been employed in all empirical studies known to us (see, e.g., Barros and Peypoch, 2007; Boussemart et al., 2003; Guironnet and Peypoch, 2007; Managi, 2003; Nakano and Managi, 2008, among others).

The profit function $\hat{\Pi}$ relative to the nonparametric technology $\hat{T}$ (Eq. (2.7) or Eq. (2.8)) is defined by $\hat{\Pi}(w,p)=\max\{p\cdot y^j-w\cdot x: j\in\mathcal{J}\}$ for all input–output price vectors $(w,p)\in\mathbb{R}^{n+p}_+$. It is now straightforward to define a dual formulation of the above nonparametric production model. If $g\in(-\mathbb{R}^n_{++})\times\mathbb{R}^p_{++}$, then $\overline{D}_{\hat{T}}(z;g)=\min_{(w,p)\geq 0}\{\hat{\Pi}(w,p)-p\cdot y+w\cdot x: p\cdot k-w\cdot h=1\}$. It follows that:

$$\overline{D}_{\hat{T}}(z;g)=\min_{(w,p)\geq 0}\left\{\max_{j\in\mathcal{J}}\{p\cdot(yj-y)-w\cdot(xj-x)\}:p\cdot k-w\cdot h=1\right\}. \tag{2.9}$$

This definition shows that this dual version of the directional distance function is a kind of shadow profit function. It can equally be used to define a mathematical program to compute the directional distance function.

To calculate the above directional distance function (2.8) in a way that guarantees non-negative outputs, one should impose the condition $y+\delta k\geq 0$ explicitly. Since (x, y) may not be in $\hat{T}$ (in this context when

[2] This directional distance function is a special case of the shortage function (Luenberger, 1992).

Table 1
Numerical example with 1 input and 1 output in two time periods.

Time	Units	Input	Output
1	1	5.0	2.0
1	2	2.5	1.0
1	3	4.0	3.0
1	4	6.5	4.0
2	1	3.0	3.0
2	2	1.0	2.0
2	3	3.0	4.0
2	4	6.5	5.0

Table 2
Numerical example: Luenberger index and proportional distance functions.

Units	$L(z^t,z^{t+1};g^t,g^{t+1})$	$D_t(z^t;g^t)$	$D_{t+1}(z^{t+1};g^{t+1})$	$D_t(z^{t+1};g^{t+1})$	$D_{t+1}(z^t;g^t)$
Specification (2.8)					
1	0.4322	0.2692	0.1667	−0.1905	0.5714
2	1.0500	0.0000	0.0000	−1.5000	0.6000
3	0.2887	0.0000	0.0000	−0.2917	0.2857
4	0.1854	0.0000	0.0000	−0.2000	0.1707
Specification (2.9)					
1	0.4322	0.2692	0.1667	−0.1905	0.5714
2	−∞	0.0000	0.0000	−∞	0.6000
3	0.2887	0.0000	0.0000	−0.2917	0.2857
4	0.1854	0.0000	0.0000	−0.2000	0.1707

computing the adjacent period distance functions $D_{t+1}(z^t;g^t)$ or $D_t(z^{t+1};g^{t+1})$), in such a case $D_{\hat{T}}(z;g)<0$ which may occasionally lead to a projection point with a negative output. Thus, a formulation for computing the directional distance function guaranteeing a traditional economic definition of non-negative outputs is:

$$\hat{D}(z;g) = sup\{\delta \in \mathbb{R} : x + \delta h \geq X\theta, y + \delta k \leq Y\theta, y + \delta k \geq 0, 1^m \cdot \theta = 1, \theta \geq 0\}. \tag{2.10}$$

This reveals that the program in Eq. (2.8) may well generate non-economic outputs when $(x,y) \notin T$.

Notice that in some more pragmatic, managerially oriented benchmarking models where, e.g., certain outputs are formulated in terms of growth rates, negative outputs resulting from a projection using a directional distance function may well be relevant (see, for instance, Portela, Thanassoulis and Simpson, 2004). In such a context, the program in Eq. (2.8) yields meaningful results.

By contrast, in standard economic production applications negative outputs have little meaning. Imposing the condition that the output translated by the directional distance function into the direction of vector k must be positive (i.e., $y+\delta k \geq 0$) solves economic meaningfulness, but it may lead to infeasible solutions for the adjacent period directional distance functions.[3] The original Färe et al. (1995) paper on the Malmquist productivity index attempts to avoid this problem by choosing a technology with a restrictive returns to scale assumption. However, Chambers and Pope (1996: 1364) rightly argue in favor of avoiding restrictive returns to scale assumptions (e.g., constant returns to scale) that are only relevant for, e.g., a representative firm supposedly to be in long-run equilibrium.

For illustration, we provide a small numerical example. Assume four units with a single input producing a single output are observed in two time periods (see Table 1). The Luenberger indicator as well as the underlying four proportional distance functions for each of these units relative to the two frontiers in both years are summarised in Table 2. Looking at the representation of the graph of both technologies in Fig. 1, the last three observations are clearly situated on the variable returns to scale frontier in each time period (whence, the zeros in the first two columns with distance functions), while the first observation is inefficient through time (whence, the positive numbers in the first two columns with distance functions). The outward shift of the technology explains the positive productivity growth revealed by the Luenberger indicator for units 3 and 4. The inefficient observation 1 moving closer to the shifting frontier also enjoys a positive productivity growth. However, the second unit illustrates the above mentioned issue: while the projection of the first period observation to the second period frontier is feasible, the reverse projection of the second period observation to the first period frontier is feasible under the standard specification, but infeasible otherwise. Indeed, the standard specification would yield a projection on the vertical segment of the frontier in the negative orthant implying that 2.5 inputs could generate a −1 output level. Imposing non-negativity of the projection point leads to an infeasibility in $D_t(z^{t+1};g^{t+1})$ yielding an undefined Luenberger indicator.

The frequency of infeasible solutions depends, among others, on the data structure, the specification of technology and the choice of direction vector (see Briec and Kerstens, in press). But, Briec and Kerstens in press show convincingly that this problem of ill-defined productivity indicators is unavoidable in general for both non-parametric and parametric technology specifications alike and that therefore the property of well-determinateness in index theory may have to be abandoned.[4] One key result is that for a given technology with at least two output dimensions and a given strictly positive direction vector, there always exists an input output vector such that the directional distance function takes the value −∞ (see Briec and Kerstens, in press, Proposition 3.1). Thus, imposing the condition $y+\delta k \geq 0$ may be just another cause of infeasibilities in empirical applications of which empirical researchers should be aware. Unfortunately, there have been few empirical studies explicitly reporting the prevalence of infeasibilities when computing, e.g., the Luenberger productivity indicator or similar productivity indices. For instance, Mukherjee et al. (2001) as well as Ray and Desli (1997) are empirical studies using a Malmquist index that do report on this problem. This lack of reporting is probably partially due to ignorance on the side of empirical researchers.

These results have also practical implications for the development of estimation procedures for technologies. For instance, attempts to correct the estimation bias in non-parametric estimators using the bootstrap currently ignore the possibility of undefined distance functions in the context of productivity indexes (see Simar and Wilson, 1999 and Tortosa-Ausina et al., 2008 for a recent empirical application), thereby introducing yet another bias in the estimates.

3. Concluding comments

This contribution has clarified a problem with the traditional way of computing the Luenberger productivity indicator. In fact, this traditional approach may lead to negative outputs when computing adjacent time period directional distance functions. To avoid this outcome, one needs to impose an additional constraint guaranteeing that the projected point yields a non-negative output. However, when

[3] Interestingly, imposing non-negativity on the resulting output projection is not necessary when using traditional Shephardian distance functions in the context of the Malmquist productivity index. For instance, when constructing the hyperbolic Malmquist productivity index (see Zofío and Lovell, 2001) using hyperbolic efficiency measures these can eventually asymptotically generate a zero output, but these can never come up with a negative output as a projection point. The additive nature of the directional distance function causes the peculiar result described here.

[4] In a similar vein, Althin (2001) is one of the few authors explicitly acknowledging that both the variable and fixed base Malmquist productivity indices may fail the determinateness test as an index.

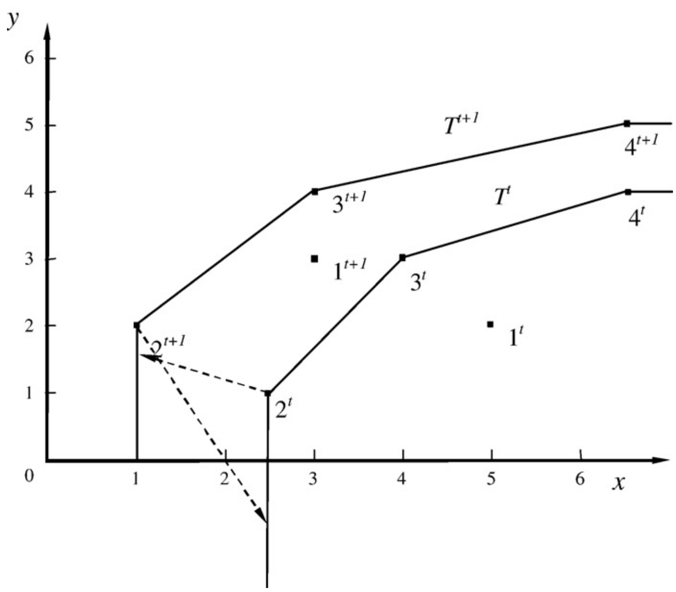

Fig. 1. Numerical example with 1 input and 1 output in two time periods.

it is binding, this additional constraint leads to infeasibilities. This is yet another potential source of ill-defined productivity indicators.

It must be stressed that infeasibilities are neither specific to the Luenberger productivity indicator nor specific to its use of the directional distance functions. As shown in Briec and Kerstens in press, infeasibilities can also occur in a variety of Malmquist productivity indices based upon Shephardian distance functions as well. Furthermore, infeasibilities can equally appear in a static efficiency setting when, for instance, evaluating the benefits from mergers (e.g., Bogetoft and Wang, 2005) or when measuring so-called super-efficiency models (e.g., Andersen and Petersen, 1993) to rank efficient units or to assess stability of the solutions. However, it is important that practitioners are aware of this infeasibility issue and why it may be logically unavoidable under certain specifications of technology. Therefore, it is recommendable to simply report any infeasibilities that happen to occur in empirical applications.

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