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THE HICKS–MOORSTEEN PRODUCTIVITY INDEX SATISFIES THE DETERMINATENESS AXIOM*

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In the literature, two types of factor productivity indices are defined as a function of a primal notion of technology. The Malmquist and the Hicks–Moorsteen productivity indices take a ratio-based approach, whereas the Luenberger and the Luenberger–Hicks–Moorsteen productivity indices follow a difference-based approach. The purpose of this note is to establish that the Hicks–Moorsteen type of productivity index, contrary to the Malmquist type of index, is well defined and satisfies the determinateness property under weak conditions on technology. This conclusion results from the fact that the underlying distance functions are always based on a feasible direction.

1 Introduction

Discrete time Malmquist productivity indices based on distance functions as general technology representations (Caves *et al.*, 1982) have been made empirically tractable by Färe *et al.* (1995) by exploiting the relation between distance functions and radial efficiency measures.[1] A problem already present in Färe *et al.* (1995) is that some of the distance functions constituting the Malmquist productivity index may well be undefined when estimated using general technologies. Unfortunately, few empirical studies explicitly report statistics on the occurrence of this infeasibility problem with the Malmquist productivity index (e.g. Glass and McKillop, 2000), thereby masking the prevalence of this problem and contributing to its neglect in the literature.[2]

Chambers (2002) introduced a more general Luenberger productivity indicator in terms of differences between directional distance functions, the latter generalizing the Shephardian distance functions.[3] Briec and Kerstens (2009) prove that infeasibilities can also occur for these directional distance

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[1]While most studies use non-parametric technologies, this same index can be computed using distance function estimates based on parametric technology specifications (e.g. Fuentes *et al.*, 2001).

[2]See Briec and Kerstens (2009) for further references reporting infeasibilities.

[3]Indicators (indices) are productivity measures based on differences (ratios).

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functions and thus the more general Luenberger productivity indicator does not satisfy the determinateness property in index theory as well. Furthermore, these authors also show that such infeasibilities may occur for both non-parametric and more traditional parametric specifications of technology alike. Determinateness is one of Fisher's (1922) original axioms and can be phrased as requiring that an index remains well defined even when one or more of its arguments become zero or infinity.[4]

Bjurek (1996, p. 310) proposes an alternative Hicks–Moorsteen index, as a ratio of Malmquist output and input indices, that has a total factor productivity (TFP) interpretation, partly to avoid the indeterminateness problem of the Malmquist index. Empirical applications of this Hicks–Moorsteen (or Malmquist TFP) index are relatively rare (e.g. Bjurek *et al.*, 1998). The Luenberger–Hicks–Moorsteen indicator (Briec and Kerstens, 2004) is a difference-based version of this ratio-based Hicks–Moorsteen index that inherits its determinateness.

However, the claim by Bjurek (1996) that the Hicks–Moorsteen index satisfies the determinateness axiom has never been proven. While this issue received little attention in the productivity index literature, it is potentially important when productivity indices are used for public policy. For instance, the implementation of incentive regulatory mechanisms in a variety of network industries (in the context of price cap regulation) would be seriously hampered when productivity change cannot be measured for some of the regulated firms (see, for example, Estache *et al.*, 2007).

The goal of this note is to prove that the Hicks–Moorsteen productivity index satisfies determinateness, thereby elucidating the underlying mechanism and conditions. Section 2 provides the basic definitions of the various distance functions and the Hicks–Moorsteen productivity index. The next section shows that this index is determinate by focusing on defining a short-run version of it. An empirical example illustrates the issues.

2 Definitions of Technology and Hicks–Moorsteen Productivity Index

We first introduce the assumptions on technology and the definitions of the distance functions. The latter provide the components for computing productivity indices.

2.1 Technology and Distance Functions

Production technology transforms inputs $x = (x_1, \ldots, x_n) \in \mathbb{R}_+^n$ into outputs $y = (y_1, \ldots, y_p) \in \mathbb{R}_+^p$. For each time period t, the production possibility set $T(t)$ summarizes the set of all feasible input and output vectors and is defined as follows:

[4]See, for example, Eichhorn (1976) and Samuelson and Swamy (1974) for conflicting views on this axiom.

$$T^t = \{(x^t, y^t) \in \mathbb{R}_+^{n+p}: x^t \text{ can produce } y^t\} \tag{1}$$

Throughout the contribution technology satisfies the following conventional assumptions: (T.1) $(0, 0) \in T^t$, $(0, y^t) \in T^t \Rightarrow y^t = 0$, i.e. no free lunch; (T.2) the set $A(x^t) = \{(u^t, y^t) \in T^t: u^t \leq x^t\}$ of dominating observations is bounded $\forall x^t \in \mathbb{R}_+^n$, i.e. infinite outputs are not allowed with a finite input vector; (T.3) T^t is closed; and (T.4) for all $(x^t, y^t) \in T^t$, $(x^t, -y^t) \leq (u^t, -v^t)$ and $(u^t, v^t) \geq 0$ implies that $(u^t, v^t) \in T^t$, i.e. fewer outputs can always be produced with more inputs, and inversely (strong disposal of inputs and outputs). Remark that we do not need the traditional convexity assumption.

Efficiency is estimated relative to production frontiers using distance or gauge functions. Distance functions are related to the efficiency measures of Farrell (1957). The Farrell efficiency measure $E_t(x^t, y^t)$ is the inverse of the Shephard distance function. In the input orientation, this measure $E_t^{\mathrm{i}}(x^t, y^t)$ indicates the minimum contraction of an input vector by a scalar λ still remaining in the technology:

$$E_t^{\mathrm{i}}(x^t, y^t) = \inf_{\lambda} \{\lambda : (\lambda x^t, y^t) \in T^t, \lambda \geq 0\} \tag{2}$$

An output efficiency measure $E_t^{\mathrm{o}}(x^t, y^t)$ searches for the maximum expansion of an output vector by a scalar θ to the production frontier, i.e. $E_t^{\mathrm{o}}(x^t, y^t) = \sup_\theta \{\theta : (x^t, \theta y^t) \in T^t, \theta \geq 1\}$.

Under constant returns to scale, input and output efficiency measures are linked: $E_t^{\mathrm{o}}(x^t, y^t) = [E_t^{\mathrm{i}}(x^t, y^t)]^{-1}$. For all $(a, b) \in \{t, t+1\}^2$, the time-related version of the Farrell input efficiency measure is given by

$$E_a^{\mathrm{i}}(x^b, y^b) = \inf_{\lambda} \{\lambda : (\lambda x^b, y^b) \in T^a\} \tag{3}$$

if there is some λ such that $(\lambda x^b, y^b) \in T^a$ and $E_a^{\mathrm{i}}(x^b, y^b) = +\infty$ otherwise. Similarly, in the output case, $E_a^{\mathrm{o}}(x^b, y^b) = \sup_\theta \{\theta : (x^b, \theta y^b) \in T^a\}$ if there is some θ such that $(x^b, \theta y^b) \in T^a$ and $E_a^{\mathrm{o}}(x^b, y^b) = -\infty$ otherwise.

2.2 The Hicks–Moorsteen Index

Following Bjurek (1996), a Hicks–Moorsteen productivity (or Malmquist TFP) index with base period t is defined as the ratio of a Malmquist output quantity index at base period t and a Malmquist input quantity index at base period t:

$$\mathrm{HM}_t(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{\mathrm{MO}_t(x^t, y^t, y^{t+1})}{\mathrm{MI}_t(x^t, x^{t+1}, y^t)} \tag{4}$$

whereby output and input quantity indices are defined as $\mathrm{MO}_t(x^t, y^t, y^{t+1}) = E_t^{\mathrm{o}}(x^t, y^t) / E_t^{\mathrm{o}}(x^t, y^{t+1})$ and $\mathrm{MI}_t(x^t, x^{t+1}, y^t) = E_t^{\mathrm{i}}(x^t, y^t) / E_t^{\mathrm{i}}(x^{t+1}, y^t)$. When the Hicks–Moorsteen productivity index is larger (smaller) than unity, it indicates productivity gain (loss).

A base period $t + 1$ Hicks–Moorsteen productivity index is defined as follows:

$$\mathrm{HM}_{t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{\mathrm{MO}_{t+1}(x^{t+1}, y^{t+1}, y^t)}{\mathrm{MI}_{t+1}(x^t, x^{t+1}, y^{t+1})} \tag{5}$$

where we have $\mathrm{MO}_{t+1}(x^{t+1}, y^{t+1}, y^t) = E^{\mathrm{o}}_{t+1}(x^{t+1}, y^t)/E^{\mathrm{o}}_{t+1}(x^{t+1}, y^{t+1})$ and $\mathrm{MI}_{t+1}(x^t, x^{t+1}, y^{t+1}) = E^{\mathrm{i}}_{t+1}(x^t, y^{t+1})/E^{\mathrm{i}}_{t+1}(x^{t+1}, y^{t+1})$.

A geometric mean of these two Hicks–Moorsteen productivity indices is (Bjurek, 1996, p. 310)

$$\mathrm{HM}_{t,t+1} = [\mathrm{HM}_t \cdot \mathrm{HM}_{t+1}]^{1/2} \tag{6}$$

where arguments of the functions are suppressed for space reasons.[5]

Notice that the denominator (numerator) of both the Malmquist output and input quantity index in base period t ($t + 1$) compares a 'hypothetical' or pseudo-observation consisting of inputs and outputs observed from different periods to a technology in period t ($t + 1$). In Bjurek's (1996, p. 310) words, the feasibility of this index is due to the fact that 'all input efficiency measures included meet the condition that the period of the technology is equal to the period of the observed output quantities' and 'all output efficiency measures included meet the condition that the period of the technology is equal to the period of the observed input quantities'.[6]

The relations between this ratio-based Hicks–Moorsteen and the more popular Malmquist productivity indices have been established in Färe *et al.* (1996).[7] An empirical study comparing both indices and reporting minor differences is Bjurek *et al.* (1998). The same study also reports on the productivity of three individual observations: for one of these, the first 10 out of 20 annual output-oriented Malmquist indices are undefined (see their Fig. 5.5).

3 The Hicks–Moorsteen Index is Determinate: Proof and Illustration by Means of a Short-run Hicks–Moorsteen Index

3.1 Construction of Feasible Farrell Technical Efficiency Measures with Fixed Input and Output Subvectors

Ouellette and Vierstraete (2004) define a short-run input-oriented Malmquist productivity index and are among the few studies reporting infeasibilities. By

[5]Note that this index can be defined in a static context to measure relative productivity between production units (see Caves *et al.*, 1982): this requires substituting the time superscripts with a unit superscript.

[6]As pointed out by a referee, while these 'hypothetical' or pseudo-observations render the Hicks–Moorsteen index feasible, these points are questionable as peers or reference points for evaluated observations compared with the case of the Malmquist index.

[7]Similar conditions relate the Luenberger–Hicks–Moorsteen and the Luenberger indicators in an additive setting (Briec and Kerstens, 2004).

focusing on the definition of a short-run (or subvector) Hicks–Moorsteen productivity index, it is possible to show the mechanism guaranteeing the well-definedness of the underlying efficiency measures.

Inspired from the construction of the Hicks–Moorsteen productivity index, we provide a general method for defining a feasible adjacent-time-period Farrell measure of technical efficiency when some inputs or outputs are fixed at their current levels in the short run.

Introducing notation, we denote $x^t = (x^{f,t}, x^{v,t})$ so that $x_i^t = x_i^{f,t}$ for $i = 1, \ldots, n_f$ and $x_i^t = x_i^{v,t}$ for $i = n_f + 1, \ldots, n$, where $n_f \in \{0, 1, \ldots, n - 1\}$. Similarly, we denote $y^t = (y^{f,t}, y^{v,t})$ so that $y_j^t = y_j^{f,t}$ for $j = 1, \ldots, p_f$ and $y_j^t = y_j^{v,t}$ for $j = p_f + 1, \ldots, p$, where $p_f \in \{0, 1, \ldots, p - 1\}$. This notation implies that there is always at least one variable input and one variable output dimension. We define the time-related input subvector Farrell measure of technical efficiency by

$$E_a^{i,f}(x^b, y^b) = \inf_{\lambda} \{\lambda : (x^{f,b}, \lambda x^{v,b}, y^b) \in T^a, \lambda \geq 0\} \tag{7}$$

if there is some λ such that $(x^{f,b}, \lambda x^{v,b}, y^b) \in T^a$ and $E_a^{i,f}(x^b, y^b) = +\infty$ otherwise, and the subvector Farrell output measure by

$$E_a^{o,f}(x^b, y^b) = \sup_{\theta} \{\theta : (x^b, y^{f,b}, \theta y^{v,b}) \in T^a, \theta \geq 1\} \tag{8}$$

if there is some θ such that $(x^b, y^{f,b}, \theta y^{v,b}) \in T^a$ and $E_a^{o,f}(x^b, y^b) = -\infty$ otherwise.

However, the above-mentioned measures are sometimes undefined, i.e. they may not obtain a finite value. Following Briec and Kerstens (2009), we say that the direction of vector $g = (h, k) \in \mathbb{R}_-^n \times \mathbb{R}_+^p$ is **infeasible** at (x, y) in period t if the affine line spanned from (x, y) in the direction of $g = (h, k)$ does not meet the production technology T^t. Suppose that $(x^b, y^b) \in T^b$. If the direction of $g = (0, -x^{v,b}, 0)$ is infeasible at (x^b, y^b) in period $t = a$, then $E_a^{i,f}(x^b, y^b) = +\infty$. A similar analysis applies to the output-oriented measure, taking $g = (0, 0, y^{v,b})$. If this direction is infeasible, then $E_a^{o,f}(x^b, y^b) = -\infty$. In such cases, one cannot compute a productivity index involving adjacent period comparisons. Similar to the Malmquist index, the resulting Hicks–Moorsteen productivity index can be undefined.

Inspired by Bjurek's approach, one can overcome this problem by constructing two other input and output time-related versions of these short-run measures. In the input-oriented case, we have

$$E_a^{i,f}(x^{f,a}, x^{v,b}, y^a) = \inf_{\lambda} \{\lambda : (x^{f,a}, \lambda x^{v,b}, y^a) \in T^a\} \tag{9}$$

if there is some λ such that $(x^{f,a}, \lambda x^{v,b}, y^a) \in T^a$ and $E_a^{i,f}(x^{f,a}, x^{v,b}, y^a) = +\infty$ otherwise. On the output side, we have

$$E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) = \sup_{\theta} \{\theta : (x^a, y^{f,a}, \theta y^{v,b}) \in T^a\} \tag{10}$$

if there is some θ such that $(x^a, y^{f,a}, \theta y^{v,b}) \in T^a$ and $E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) = -\infty$ otherwise.

In the following, we say that input factors are essential if $(x^t, y^t) \in T^t$ and $y^t \neq 0$ implies that $x_i^t > 0$ for $i = 1, \ldots, n$.

Lemma 1: Assume that technology satisfies (T.1)–(T.4) and that the inputs are essential. For $(a, b) \in \{t, t+1\}^2$, *if* $x^{v,b} \neq 0$, *then* $0 < E_a^{i,f}(x^{f,a}, x^{v,b}, y^a) < +\infty$.

Proof: Fix $\lambda^* = \max\{x_i^{v,a}/x_i^{v,b} : i = n_f + 1, \ldots, n, x_i^{v,b} > 0\}$ and let us consider the vector $(x^{f,a}, \lambda^* x^{v,b}, y^a)$. Elementary calculus indicates that $(x^{f,a}, \lambda^* x^{v,b}) \geq (x^{f,a}, x^{v,a}) = x^a$. From the strong disposability assumption, we deduce that $(x^{f,a}, \lambda^* x^{v,b}) \in T^a$ and consequently $E_a^{i,f}(x^{f,a}, x^{v,b}, y^a) < +\infty$. Moreover, since the factors are essential and $x^{v,b} \neq 0$, the second inequality follows. ■

Lemma 2: Assume that technology satisfies (T.1)–(T.4). For $(a, b) \in \{t, t+1\}^2$, *if* $y^{v,a} \neq 0$ *and* $y^{v,b} \neq 0$, *then* $0 < E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) < +\infty$.

Proof: Fix $\theta^* = \min\{y_j^{v,a}/y_j^{v,b} : j = p_f + 1, \ldots, p, y_j^{v,b} > 0\}$ and let us consider the vector $(x^a, y^{f,a}, \theta^* y^{v,b})$. Thus, $(y^{f,a}, \theta^* y^{v,b}) \leq (y^{f,a}, y^{v,a}) = y^a$ and from the strong disposability assumption we deduce that $(y^{f,a}, \theta^* y^{v,b}) \in T^a$ and $E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) < +\infty$. Moreover, since $y^{v,a} \neq 0$ and $y^{v,b} \neq 0$, we deduce that $E_a^{o,f}(x^a, y^{f,a}, y^{v,b}) > 0$. ■

These results immediately translate into our main result with respect to the Hicks–Moorsteen productivity index.

Proposition 1: If for all $(a, b) \in \{t, t+1\}^2$, we have $y^a \neq 0$ and $y^b \neq 0$, then the Hicks–Moorsteen productivity index (6) is well defined.

Proof: Since there is no free lunch, the result follows directly from taking $n_f = p_f = 0$ in Lemmas 1 and 2. ■

Figure 1 shows an input isoquant from technology in period t and two observations $(x^{f,t}, x^{v,t})$ and $(x^{f,t+1}, x^{v,t+1})$. It is clearly impossible to achieve the distance from $(x^{f,t+1}, x^{v,t+1})$ to the input isoquant in period t in the direction of the variable input dimension. In contrast, when creating the pseudo-observation $(x^{f,t}, x^{v,t+1})$ a distance can be measured relative to this isoquant.

3.2 A Determinate Hicks–Moorsteen Productivity Index with Subvectors

A base period t short-run Hicks–Moorsteen feasible productivity index is defined as follows:

$$\mathrm{HM}_t^f(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{\mathrm{MO}_t^f(x^t, y^t, y^{t+1})}{\mathrm{MI}_t^f(x^t, x^{t+1}, y^t)} \tag{11}$$

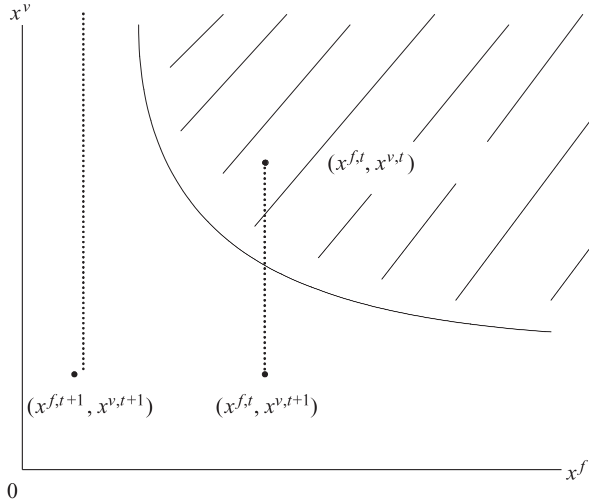

FIG. 1 Feasible Subvector Input Efficiency Measure

where, by definition, $\mathrm{MO}_t^f(x^t, y^t, y^{t+1}) = E_t^{o,f}(x^t, y^{f,t}, y^{v,t})/E_t^{o,f}(x^t, y^{f,t}, y^{v,t+1})$ and $\mathrm{MI}_t^f(x^t, x^{t+1}, y^t) = E_t^{i,f}(x^{f,t}, x^{v,t}, y^t)/E_t^{i,f}(x^{f,t}, x^{v,t+1}, y^t)$.

A base period $t + 1$ short-run Hicks–Moorsteen feasible productivity index is defined as follows:

$$\mathrm{HM}_{t+1}^f(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{\mathrm{MO}_{t+1}^f(x^{t+1}, y^{t+1}, y^t)}{\mathrm{MI}_{t+1}^f(x^t, x^{t+1}, y^{t+1})} \tag{12}$$

where $\mathrm{MO}_{t+1}^f(x^{t+1}, y^{t+1}, y^t) = E_{t+1}^{o,f}(x^{t+1}, y^{f,t+1}, y^{v,t})/E_{t+1}^{o,f}(x^{t+1}, y^{f,t+1}, y^{v,t+1})$ and $\mathrm{MI}_{t+1}^f(x^t, x^{t+1}, y^{t+1}) = E_{t+1}^{i,f}(x^{f,t+1}, x^{v,t}, y^{t+1})/E_{t+1}^{i,f}(x^{f,t+1}, x^{v,t+1}, y^{t+1})$. A geometric mean of these two feasible short-run Hicks–Moorsteen productivity indices is $\mathrm{HM}_{t,t+1}^f = [\mathrm{HM}_t^f \cdot \mathrm{HM}_{t+1}^f]^{1/2}$, where the arguments of the functions have again been suppressed to save space and the index remains determinate because it uses feasible efficiency measures of the type (9) and (10).

In contrast, the following variation on this short-run Hicks–Moorsteen productivity index is not well defined. For reasons of space, we limit ourselves to only developing the base period t case. A base period t short-run Hicks–Moorsteen productivity index that is infeasible is defined as follows:

$$\mathrm{HM}_t^{f'}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{\mathrm{MO}_t^{f'}(x^t, y^t, y^{t+1})}{\mathrm{MI}_t^{f'}(x^t, x^{t+1}, y^t)} \tag{13}$$

where we have $\mathrm{MO}_t^{f'}(x^t, y^t, y^{t+1}) = E_t^{o,f}(x^t, y^{f,t}, y^{v,t})/E_t^{o,f}(x^t, y^{f,t+1}, y^{v,t+1})$ and $\mathrm{MI}_t^{f'}(x^t, x^{t+1}, y^t) = E_t^{i,f}(x^{f,t}, x^{v,t}, y^t)/E_t^{i,f}(x^{f,t+1}, x^{v,t+1}, y^t)$. The infeasibility results from using efficiency measures of the type (7) and (8).

A comparison with the previous version index shows that the feasibility of the Malmquist output quantity index is achieved by comparing period t inputs and outputs with period t inputs and fixed outputs and period $t + 1$ variable outputs. In contrast, the latter infeasible case compares period t inputs and outputs with period t inputs and period $t + 1$ fixed and variable outputs. Thus, by simply keeping the fixed outputs firmly in the previous period, the output efficiency measures can be evaluated with respect to the resulting pseudo-observation. This logic is clearly in line with the basic intuitions cited above from Bjurek (1996).

4 Empirical Illustration

By way of empirical illustration, we revisit the estimation of macroeconomic productivity gains for 20 Organization of Economic Cooperation and Development (OECD) countries over the period 1974–97 analysed in Boussemart *et al.* (2003). Technology is defined as a simple single output, gross domestic product evaluated at 1990 prices, which is produced by two inputs: capital and labour. Capital is the fixed input in this analysis. More details on these data are available in Boussemart *et al.* (2003). In particular, we compare the traditional long-run as well as the feasible short-run Hicks–Moorsteen productivity index (based on components (11) and (12)) to a short-run version of a traditional input-oriented Malmquist productivity index. To this purpose, we first turn to a definition of an input-oriented Malmquist productivity index and its short-run version.

A geometric mean input-oriented Malmquist productivity index is defined as follows:

$$M^{\mathrm{i}}_{t,t+1} = \left[\frac{E^{\mathrm{i}}_t(x^t, y^t)}{E^{\mathrm{i}}_t(x^{t+1}, y^{t+1})} \cdot \frac{E^{\mathrm{i}}_{t+1}(x^t, y^t)}{E^{\mathrm{i}}_{t+1}(x^{t+1}, y^{t+1})} \right]^{1/2} \tag{14}$$

The first (second) ratio is an index for base period t ($t + 1$), whereby values below (above) unity reveal productivity growth (decline). Its short-run counterpart is

$$M^{\mathrm{i},f}_{t,t+1} = \left[\frac{E^{\mathrm{i},f}_t(x^{f,t}, x^{v,t}, y^t)}{E^{\mathrm{i},f}_t(x^{f,t+1}, x^{v,t+1}, y^{t+1})} \cdot \frac{E^{\mathrm{i},f}_{t+1}(x^{f,t}, x^{v,t}, y^t)}{E^{\mathrm{i},f}_{t+1}(x^{f,t+1}, x^{v,t+1}, y^{t+1})} \right]^{1/2} \tag{15}$$

In line with our assumptions (T.1)–(T.4), we use a non-parametric specification of technology imposing either constant (CRS) or variable (VRS) returns to scale to measure the efficiency measures underlying both the traditional long-run as well as the short-run Hicks–Moorsteen and input-oriented Malmquist productivity indices (see Bjurek (1996) or Ouellette and Vierstraete (2004) for details).

Empirical results are summarized in Table 1. It is structured as follows: the upper (lower) part contains results for the input-oriented Malmquist

TABLE 1
INPUT-ORIENTED MALMQUIST VERSUS HICKS–MOORSTEEN INDICES: DESCRIPTIVE STATISTICS

	Input-oriented Malmquist CRS	*Input-oriented Malmquist VRS*	*Subvector input-oriented Malmquist CRS*	*Subvector input-oriented Malmquist VRS*
Mean	0.9933	0.9915	0.9913	0.9933
Standard deviation	0.0234	0.0246	0.0367	0.0337
Minimum	0.8590	0.8758	0.7856	0.8256
Maximum	1.0954	1.0840	1.1650	1.1425
No. infeasible observations	0	23	32	139
% infeasible observations	0.00%	4.79%	6.67%	28.96%
	Hicks–Moorsteen CRS	*Hicks–Moorsteen VRS*	*Subvector Hicks–Moorsteen CRS*	*Subvector Hicks–Moorsteen VRS*
Mean	1.007267	1.008600	1.001667	1.001667
Standard deviation	0.023819	0.023812	0.026287	0.026287
Minimum	0.912889	0.914396	0.908226	0.908226
Maximum	1.164156	1.167658	1.162632	1.162632
No. infeasible observations	0	0	0	0
% infeasible observations	0.00%	0.00%	0.00%	0.00%

(Hicks–Moorsteen) productivity index; the left (right) part contains results for the traditional long-run (short-run) versions of the same indices. We just report aggregate descriptive statistics: mean, standard deviation, minimum and maximum.

Turning to the empirical results in Table 1, we first notice that the traditional long-run input-oriented Malmquist and Hicks–Moorsteen productivity indices yield a similar qualitative message of moderate productivity growth in all OECD countries. For instance, the ones computed on a CRS technology have a product–moment correlation of –0.7398 (see also Bjurek *et al.* (1998) for further comparisons between the two indices).[8] Second, turning to the issue of infeasibilities, notice that, once we drop the CRS assumption or the long-term perspective, infeasibilities start appearing in the input-oriented Malmquist productivity index (from about 5 per cent to 29 per cent of the sample). Just to provide some information at the level of individual countries, in the latter case three and four countries have no respectively some results for the Malmquist over the whole period. Needless to say, no such phenomenon can be observed in the case of the Hicks–Moorsteen productivity index. Obviously, the occurrence of infeasible solutions complicates the comparison between the two types of indices.

[8]Notice that in the case of the input-oriented Malmquist (Hicks–Moorsteen) productivity index values below (above) unity reveal productivity growth.

5 Conclusions

This note demonstrates that the Hicks–Moorsteen productivity index satisfies determinateness under weak conditions on technology. In contrast, the more popular Malmquist productivity index does not meet this demand. The same result can be transposed to the difference-based counterparts of both these indices. These two types of productivity indices are thus structurally different, even though empirical differences have sometimes been found to be minor (e.g. Bjurek *et al.*, 1998).

We expect the Hicks–Moorsteen productivity index to gain in popularity in future empirical work, especially when infeasible solutions are simply unacceptable (e.g. in incentive-based regulatory mechanisms where the efficiency requirements of price caps must be determined under all circumstances to avoid gaming the regulator). It remains an open question whether solutions suggested in the literature to remedy this problem (e.g. Tulkens and Vanden Eeckaut, 1995) prove to be viable.

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