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Temporal technical and profit efficiency measurement: Definitions, duality and aggregation results

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Abstract

The shortage function, an important tool in production theory, measures potential increases in outputs and decreases in inputs for a given direction g at a given date. To develop a temporal version of technical efficiency measurement, we introduce the concept of a temporal shortage function. This temporal efficiency measure is easily computed using linear programming. We also establish a duality result stating that the temporal profit function and the temporal shortage function are dual to one another. This result has two consequences. First, one can derive a shadow price path via the shadow prices of the temporal shortage function. Second, transposing the classic Farrell inefficiency decomposition, temporal profit efficiency is decomposed into temporal technical and temporal allocative efficiency components. Finally, in line with the recent literature on aggregation over firms, this contribution treats the possibilities and limits of the aggregation of efficiency measures over time.

Keywords: Temporal shortage function; Temporal profit function; Aggregation over time

1. Introduction

The main purpose of this article is the introduction of time into the analysis of technical efficiency and duality results. The suggested methodology looks at technical efficiency from the new angle of global performance, which means that we consider efficiency measurement over a given time period while ignoring the possibility of technological change altogether. The traditional efficiency analysis is static and evaluates the performance of decision-making units (DMU) at a given date. In our framework, which builds upon rigorous axioms and non-parametric methods, we show that the global performance over time corresponds to the concept of average performance. This notion of average performance will be made more precise in the contribution.

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While economics as a discipline has always devoted much attention to technical change, it has only fairly recently been recognized that ignoring inefficiencies may well bias the measurement of total factor productivity (e.g., Nishimizu and Page, 1982). This is due to the recent interest in efficiency benchmarking based upon parametric and non-parametric production and value frontiers (see Lovell, 1993 for a survey). This contribution can be interpreted as thinking this efficiency literature into its extreme consequences: we investigate efficiency over time while completely ignoring the possibility of frontier changes. We do not claim that technological change is of no importance. Rather, we maintain that it may be useful to abstract from frontier changes to obtain a more precise idea of efficiency over time. Thinking these issues through fills a gap in the literature and adds a new empirical tool to the analysis of industries where the role of technological changes is a priori very limited (e.g., due to investments in large indivisible infrastructures embodying technological change) and the main focus is on managing the performance over time with respect to a given technology.

The inspirations for this work are the books by Färe (1988), Färe and Grosskopf (1996), and Sengupta (1995, 2003), which extends the concept of efficiency into an intertemporal context. Jaenicke (2000) is one of the first empirical applications of this model, while also integrating the use of intermediate production factors in agriculture. Silva and Stefanou (2003) extend in a dynamic way the traditional framework while taking into account the fixity of inputs and the investment decision. Indeed, they recover technological information from dynamic cost minimizing behavior without imposing a parametric functional form on technology and while accounting for adjustment-costs.

In particular, we invoke the general assumption of temporal separability of technologies between successive time periods. While technologies in each time period may well be different from one another, we consider the Cartesian product of all technologies in all time periods simultaneously. Notice that we distinguish conceptually between these technologies per time period, but we do not focus on shifts in these successive frontiers, but rather on the relative efficiency of units with respect to these successive frontiers. Clearly, since we do not allow for linkages between optimal decisions between time periods, our models are only dynamic in a limiting sense. Therefore, given our focus on the relative efficiency of units compared to successive frontiers over time, we propose to use the terminology "temporal efficiency measure".[1] By contrast, the books by Färe (1988), Färe and Grosskopf (1996), and Sengupta (1995, 2003) do allow for time substitution, i.e., the timing of inputs utilization. In addition, it is worthwhile mentioning that various other dynamic phenomenon, like adjustment costs (i.e., adjustment of short run input decisions to attain the optimal temporal trajectory in response to, e.g., output and input price fluctuations as a model of learning behavior), have been studied in Sengupta (1992, 1999, 2003), among others.

While modern duality theory goes back to Shephard (1953, 1970), McFadden (1978) and Diewert (1982), it is the recent introduction of the shortage function defined on the graph of technology that enabled defining a duality in terms of the profit function (see Chambers et al. (1998) or Färe and Grosskopf (2000) for proofs of duality between shortage and profit functions). Our contribution focuses on the most general value function, namely the profit function. The use of these recent tools in the temporal analysis of efficiency over time is—to the best of our knowledge- an original contribution to the literature on applied production theory.

In particular, this contribution innovates on the following points. First, we integrate a temporal dimension in the recently proposed efficiency measures of Luenberger (1992, 1995) and Chambers

[1] Tulkens and Vanden Eeckaut (1995) introduced the notion of an intertemporal technology. However, this implies ignoring the time dimension of technologies altogether and amalgamating all observations irrespective of their time dimensions in the construction of a single production frontier. We maintain the time dimension of technologies, because we focus on efficiency measurement relative to each technology over time. Thus, our focus is on the time path of efficiency. However, since we maintain temporal separability throughout, we avoid the use of the word dynamic or even intertemporal.

et al. (1996, 1998). Then, we develop a duality result relating a temporal profit function and this temporal efficiency measure. To this purpose, we define a technological path in terms of prices. Starting from this technological path and the temporal profit function, we recover the temporal production technology. Then, we show that we can obtain a path of shadow prices. Finally, this contribution treats the possibilities and limits of the aggregation of efficiency measures over time, in accordance with some recent articles on the aggregation over firms within a given sector.

The next section defines the temporal graph of technology and exposes the axioms underlying this same temporal technology. In Section 3 a temporal efficiency measure is build starting from the static shortage function. In a fourth section, we establish duality between the temporal profit function and the temporal shortage function. Next, we develop some aggregation results over time. Finally, Section 6 concludes and suggests some plausible extensions.

2. A temporal technology defined as a temporal product of technologies

In a discrete time framework, the input–output space is denoted $\mathop{\times}\limits_{t=1}^{\mathrm{T}}(R_+^{N+M}) = (R_+^{N+M})^{\mathrm{T}}$. It consists of all sequences of dated inputs and outputs of the form

$$(X, Y) = \mathop{\times}\limits_{t=1}^{\mathrm{T}}(x^t, y^t) = (x^t, y^t)_{t=1}^{\mathrm{T}} = (x_1^t, \ldots, x_N^t, y_1^t, \ldots, y_M^t)_{t=1}^{\mathrm{T}}.$$

Since the assumption of temporal separability is maintained throughout the paper, this amounts to working on a multidimensional technology raised to the Cartesian product of all time periods. This work is very similar to the work of Färe (1988, Sections 8.1–8.2) and Färe and Grosskopf (1996, Section 6.1).

2.1. Temporal graph of technology

The temporal graph of technology involves all possible inputs and outputs at each date and is defined by: $\mathcal{GR} = \mathop{\times}\limits_{t=1}^{\mathrm{T}} GR^t$. In fact, this simply boils down to the product of a series of graphs of technology in the static case.

Definition 1. A technological path is any vector $(x^t, y^t)_{t=1}^{\mathrm{T}} = (X, Y) \in \mathcal{GR}$. The trajectory $(X, Y) \in \mathcal{GR}$ represents all input and output vectors such that x^t can produce y^t at date t.

Fig. 1 (similar to Färe (1988, Fig. 8.1) illustrates a possible configuration of the temporal graph of technology on an interval $\{1, \ldots, T\}$ for $n = m = 1$. It shows the evolution of a single input and output technology over time, where time is represented on a third axis. Clearly, the technologies in each time period can be different from one another, but they are unrelated to one another due to the temporal separability assumption. By contrast, Färe (1988), Färe and Grosskopf (1996) and Sengupta (2003) explicitly study time substitution of inputs, i.e., the utilization of inputs over time.

2.2. Axioms of the temporal production technology

The axioms imposed on the temporal set of production possibilities are the following:

GR1: $(0,0) \in \mathcal{GR}, (0, Y) \in \mathcal{GR} \Rightarrow Y = 0$.
GR2: $\mathcal{GR}(Y) = \{(X, Y') \in \mathcal{GR}, | Y' \leqslant Y\}$ is bounded $\forall Y \in (R_+^M)^{\mathrm{T}}$.
GR3: If $\lambda = (\lambda^1, \lambda^2, \ldots, \lambda^{\mathrm{T}})$, if $(X, Y) \in \mathcal{GR}$, $(\lambda X, Y) \in \mathcal{GR}\ \forall \lambda \geqslant 1^{\mathrm{T}}$.
GR4: $\forall (X, Y) \in \mathcal{GR}$, if $(\hat{X}, Y) \geqslant (X, Y) \Rightarrow (\hat{X}, Y) \in \mathcal{GR}$.
GR5: $\mathcal{GR}$ is closed.
GR6: If $\theta = (\theta^1, \theta^2, \ldots, \theta^{\mathrm{T}})$, if $(X, Y) \in \mathcal{GR}$, then $(X, \theta Y) \in \mathcal{GR}$, $\forall \theta \in [0, 1]^{\mathrm{T}}$.
GR7: $\forall (X, Y) \in \mathcal{GR}$, if $(X, \hat{Y}) \leqslant (X, Y) \Rightarrow (X, \hat{Y}) \in \mathcal{GR}$.
GR8: $\mathcal{GR}$ is convex.

By analogy to the static production axioms, we impose traditional regularity conditions such as possibility of inaction and no free lunch (GR1), as well as boundedness (GR2), closedness (GR5), and convexity of the technology (GR8). Furthermore, we allow for strong input (GR4) or output (GR7) disposability. Alternatively, it is possible to impose

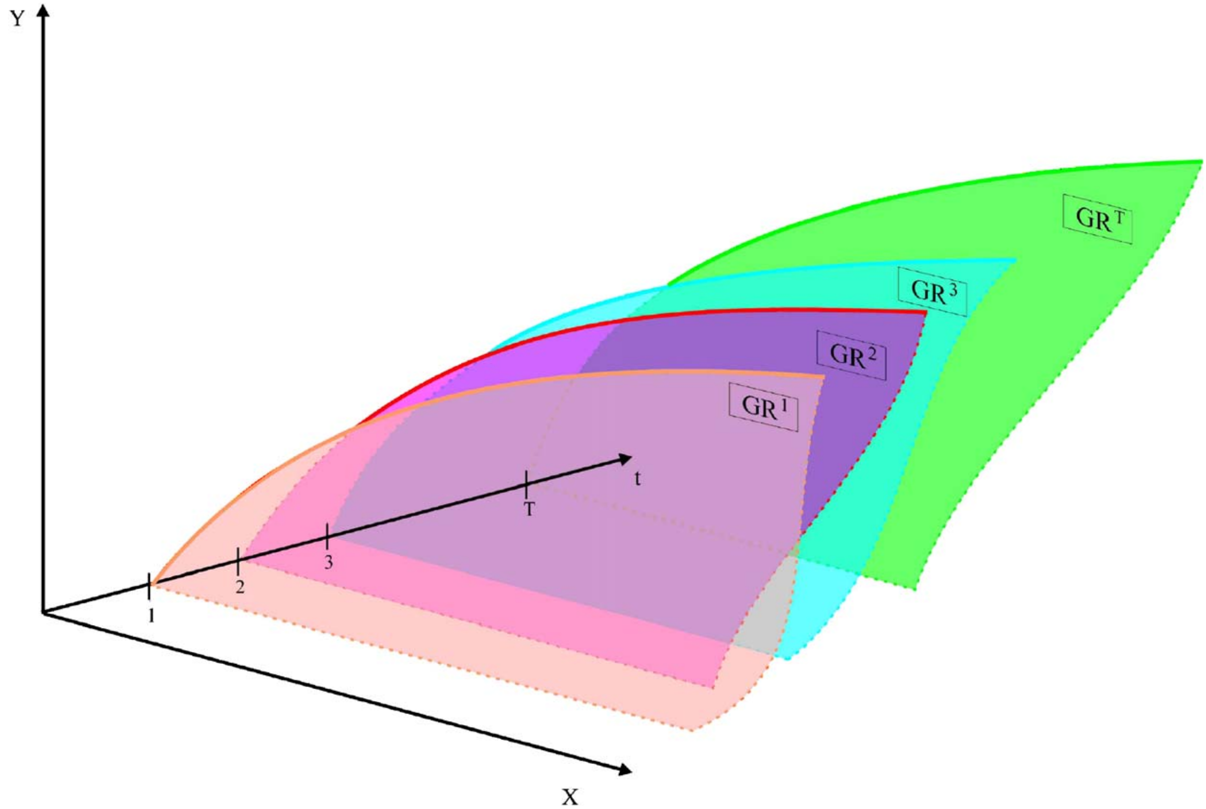

Fig. 1. Temporal graph of technology.

weak input (GR3) or output (GR6) disposability.[2] A rather similar axiomatic structure is discussed in Farë (1988, pp. 119–120).

3. Temporal technical efficiency measured by the temporal shortage function: Definition, properties and estimation

In this section, we first define the shortage function introduced by Luenberger (1992, 1995).[3] Before presenting the traditional static as well as the new temporal version of the shortage function, we establish a lemma that proves useful in the remainder of this section.

Lemma 1. *If $A^1, A^2, \ldots, A^{\mathrm{T}}$ are T subsets of R_+^{N+M}, and if $f_1, f_2, \ldots, f_T$ are T functions such that $\forall t \in \{1, \ldots, T\}$, f_t is defined on A^t in R_+^{N+M}, then the following property holds*:

$\forall t, f_t : A^t \to R$ *and* $A^t \subset R_+^{N+M}$

$$\inf\left\{\sum_{t=1}^{\mathrm{T}} f_t(x^t), (x^1, \ldots\ldots, x^{\mathrm{T}}) \in A^1 \times \cdots\cdots \times A^{\mathrm{T}}\right\} = \sum_{t=1}^{\mathrm{T}} \inf\{f_t(x^t), x^t \in A^t\}.$$

[2] Notice that it is possible to formulate an axiom of strong (weak) disposability in the input and output dimensions simultaneously by combining axioms GR4 and GR7 (GR3 and GR6). However, we refrain from doing so, because it is also possible to combine strong input disposability with weak output disposability, or the reverse. Therefore, this way of structuring the axioms opens up more general specifications of the temporal production technology.

[3] Chambers et al. (1998) rename it a directional distance function.

Proof. The function $f(x^1, \ldots, x^{\mathrm{T}}) = \sum_{t=1}^{\mathrm{T}} f_t(x^t)$ is separable, since is defined on a Cartesian product. Since it is optimized on the Cartesian product $\underset{t=1}{\overset{\mathrm{T}}{\times}} A^t$, it follows immediately that the inf of the sum of the functions is the sum of the inf of the functions. □

3.1. Temporal shortage function: A definition

To define the static shortage function introduced by Luenberger (1992, 1995) and Chambers et al. (1996, 1998), note that the vector $g \in (-R_+^N) \times R_+^M$ and that $g^t = (-g_i^t, g_o^t)$, the vector g^t representing a direction in the input–output space at date t.

Definition 2. If GR^t is a technology at t satisfying GR1–GR8 and $(x^t, y^t) \in GR^t$ is a vector of inputs and outputs, then the static shortage function is defined as

$$S(x^t, y^t, g^t) = \max_{\delta^t}\{\delta^t | (x^t, y^t) + \delta^t g^t \in GR^t\}.$$

Notice that the static shortage function projects each input–output vector in the direction of g onto the boundary of the technology. The value of the function δ^t is positive or null depending on whether the vector is situated in the interior or on the boundary of technology.

Building upon this definition, we seek to define a temporal measure of technical efficiency that summarizes the sequence of distances between the technological path of a production unit and the temporal production technology for a given a path of direction. This directional path is denoted $G = (g^1, \ldots, g^{\mathrm{T}}) \in \underset{t=1}{\overset{\mathrm{T}}{\times}}[(-R_+^N) \times R_+^M] = [(-R_+^N) \times R_+^M]^{\mathrm{T}}$, i.e., a direction used by the decision maker to improve efficiency. From an economic point of view, this directional path G and each of its vector elements g can be interpreted as reference directions for the producer over time. Thus, the producer seeks to adjust its actual production path over time according to a direction that also moves over time. In brief, G provides the directions for evaluating the technical efficiency index measuring the distance between the observed technological path (X, Y) and the efficient path.

Definition 3. If $\mathscr{GR}$ is a temporal production technology satisfying GR1–GR8, (X, Y) is an input–output path in $\mathscr{GR}$, and $\delta = (\delta^1, \delta^2, \ldots, \delta^{\mathrm{T}})$, then the temporal shortage function is defined as follows:

$$S(X, Y, G) = \max_{\delta}\left\{\sum_{t=1}^{\mathrm{T}} \frac{\delta^t}{T} | (X, Y) + \delta G \in \mathscr{GR}\right\}.$$

This amounts to looking for an arithmetic mean of simultaneous reductions in inputs and expansions in outputs into a path of direction G such that an observed input–output path (X, Y) is projected onto the boundary of the temporal production technology.

We can immediately prove the following proposition regarding this temporal shortage function.

Proposition 1. *If $\mathscr{GR}$ is a temporal production technology satisfying GR1–GR8, (X, Y) is an input–output path in $\mathscr{GR}$, and $\delta = (\delta^1, \delta^2, \ldots, \delta^{\mathrm{T}})$, then the temporal shortage function $S(X, Y, G)$ can be written as follows:*

$$\begin{aligned} S(X, Y, G) &= \max_{\delta}\left\{\sum_{t=1}^{\mathrm{T}} \frac{\delta^t}{T} | (X, Y) + \delta G \in \mathscr{GR}\right\} \\ &= \max_{\delta}\left\{\sum_{t=1}^{\mathrm{T}} \frac{\delta^t}{T} | (x^t, y^t) + \delta^t g^t \in GR^t\right\} \\ &= \frac{1}{T}\sum_{t=1}^{\mathrm{T}} \max_{\delta^t}\{\delta^t | (x^t, y^t) + \delta^t g^t \in GR^t\} \\ &= \frac{1}{T}\sum_{t=1}^{\mathrm{T}} S(x^t, y^t, g^t). \end{aligned}$$

Proof. It follows directly from the application of Lemma 1. □

Thus, the temporal shortage function is easily calculated, because it simply corresponds to the arithmetic mean of the static shortage functions over the whole time horizon. The value of the components of the vector δ is again positive or zero depending on whether the evaluated point is in the interior or on the boundary of technology in any given time period. Fig. 2 illustrates the temporal shortage function for $n = m = 1$ over the period $\{1, \ldots, T\}$. The dashed line represents

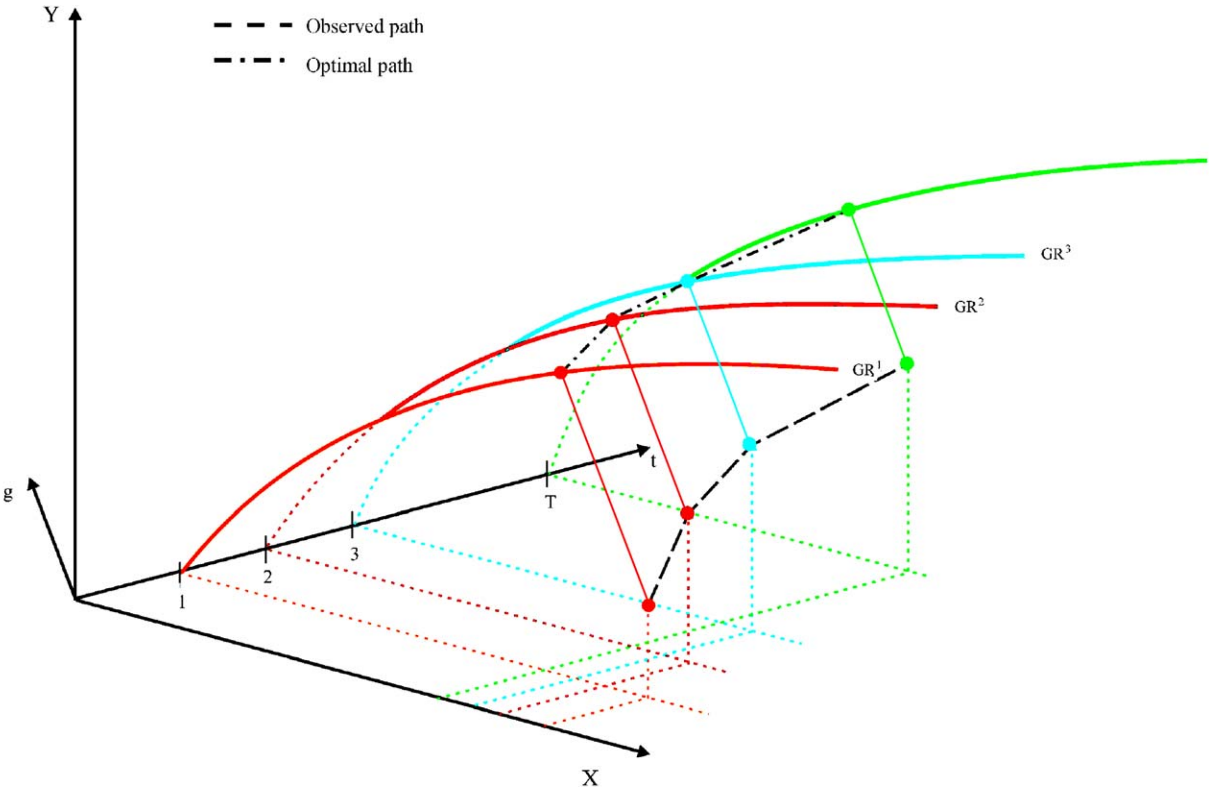

Fig. 2. Temporal shortage function: observed and optimal paths.

the observed path over time. In each time period, the observation is clearly situated below the frontier. The temporal shortage function is simply a vector of distances to each of the respective boundaries of the technologies evolving over time (represented by the dash dot line).

3.2. Properties of the temporal shortage function

The temporal shortage function, as a summary measure of efficiency over time, satisfies a number of attractive properties summarized in the following proposition:

Proposition 2. *Assume that $\mathcal{GR}$ satisfies axioms $GR1$–$GR8$, then $S(X,Y,G):\overset{T}{\underset{t=1}{\times}}(R_+^{N+M})\to R$ satisfies the following properties*:

(1) *If $\mathcal{GR}$ is convex, then $S(X,Y,G)$ is concave in relation to (X,Y).*

(2) $S((X,Y)+\alpha G,G)$
$$= S(X,Y,G)-\bar{\alpha}\quad \forall\alpha\in\overset{T}{\underset{t=1}{\times}}(R_+^{N+M}).$$

(3) *If* $(X,Y)\in\mathcal{GR}\Rightarrow S(X,Y,G)\geqslant 0$.

(4) $S(X,Y,\mu G)=(1/\mu)S(X,Y,G)\quad\forall\mu\geqslant 0$.

(5) $\forall(X,Y),(X',Y')\in\mathcal{GR}$, *if* $(-X',Y')\geqslant(-X,Y)$ $\Rightarrow S(X',Y')\leqslant S(X,Y)$.

Proof. (1) Let $(X,Y)\in\mathcal{GR}$ and $(X',Y')\in\mathcal{GR}$. If $\mathcal{GR}$ is convex, we have $\theta(X,Y)+(1-\theta)(X',Y')\in\mathcal{GR}\Rightarrow\theta(x^t,y^t)+(1-\theta)(x'^t,y'^t)\in GR^t$. So, $\forall t$, GR^t is convex.

$$\Rightarrow S(\theta(x^t,y^t,g^t)+(1-\theta)(x'^t,y'^t,g^t))$$

$$\geqslant\theta S(x^t,y^t,g^t)+(1-\theta)S(x'^t,y'^t,g^t)$$

$$\Rightarrow\frac{1}{T}\sum_{t=1}^{T}S(\theta(x^t,y^t,g^t)+(1-\theta)(x'^t,y'^t,g^t))$$

$$\geqslant \theta \frac{1}{T} \sum_{t=1}^{T} S(x^t, y^t, g^t) + (1-\theta) \frac{1}{T} \sum_{t=1}^{T} S(x'^t, y'^t, g^t)$$

$$\Rightarrow S(\theta(X, Y, G) + (1-\theta)(X', Y', G)) \geqslant \theta S(X, Y, G) + (1-\theta) S(X', Y', G).$$

Hence, $S(X, Y, G)$ is concave in relation to (X, Y).

(2) Let $\alpha = (\alpha^1, \alpha^2, \ldots, \alpha^T)$ and $G = (g^1, g^2, \ldots, g^T)$. We denote $\alpha G = (\alpha^1 g^1, \ldots, \alpha^T g^T)$.

$$\begin{aligned} S((X, Y) + \alpha G, G) &= \frac{1}{T} \sum_{t=1}^{T} S((x^t, y^t) + \alpha^t g^t, g^t) \\ &= \frac{1}{T} \sum_{t=1}^{T} (S(x^t, y^t, g^t) - \alpha^t) \\ &= \frac{1}{T} \sum_{t=1}^{T} S(x^t, y^t, g^t) - \frac{1}{T} \sum_{t=1}^{T} \alpha^t \\ &= S(X, Y, G) - \bar{\alpha}. \end{aligned}$$

Remark that if $\forall t$, $\alpha^t = \alpha$, then we have

$$\Rightarrow \bar{\alpha} = \frac{1}{T} \sum_{t=1}^{T} \alpha^t = \frac{1}{T} T\alpha = \alpha \Rightarrow S(X + \alpha G, Y, G) = S(X, Y, G) - \alpha.$$

(3) Let $(X, Y) \in \mathcal{GR} \Leftrightarrow (x^t, y^t) \in GR^t$, $\forall t$. According to Luenberger (1992) and Chambers et al. (1996, 1998): $\forall t, S(x^t, y^t, g^t) \geqslant 0 \Rightarrow 1/T \sum_{t=1}^{T} S(x^t, y^t, g^t) \geqslant 0 \Rightarrow S(X, Y, G) \geqslant 0$. Then, if $(X, Y) \in \mathcal{GR} \Rightarrow S(X, Y, G) \geqslant 0$.

(4) Let $\mu^t = \mu$, $\forall t$.

$$\begin{aligned} S(X, Y, \mu G) &= \frac{1}{T} \sum_{t=1}^{T} S(x^t, y^t, \mu g^t) \\ &= \frac{1}{T} \sum_{t=1}^{T} \frac{1}{\mu} S(x^t, y^t, g^t) \\ &= \frac{1}{\mu} \left(\frac{1}{T} \sum_{t=1}^{T} S(x^t, y^t, g^t) \right) \\ &= \frac{1}{\mu} S(X, Y, G). \end{aligned}$$

Thus, the shortage function is homogeneous of degree -1 in relation to G.

(5) Suppose that $(-X', Y') \geqslant (-X, Y)$, this involves that $(-x'^t, y'^t) \geqslant (-x^t, y^t)$, $\forall t = 1, \ldots, T$. According to Chambers et al. (1996, 1998), we have

$$\begin{aligned} S(x'^t, y'^t, g^t) \leqslant S(x^t, y^t, g^t) &\Rightarrow \frac{1}{T} \sum S(x'^t, y'^t, g^t) \\ &\leqslant \frac{1}{T} \sum S(x^t, y^t, g^t) \\ &\Rightarrow S(X', Y', G) \leqslant S(X, Y, G). \qquad \square \end{aligned}$$

These properties of the temporal shortage function can be briefly clarified as follows. If the temporal technology is convex, then the temporal shortage function is concave in relation to the evaluated technological path. The second property corresponds to the static translation homotheticity property and states that the value of the temporal shortage function of an observed technological path translated by αG equals the value of the shortage function of the technological path (X, Y) minus the mean value of α. Property 3 shows that the temporal shortage function provides a total description of the temporal technology. Moreover, according to property 4 it is homogeneous of degree -1 in relation to G. This implies that when the directional path is multiplied by a number, then the function is reduced in the opposite proportion. Finally, the temporal shortage function satisfies a weak monotonicity property, i.e., for any technological path that weakly dominates another path (X, Y), the value of the function is weakly lower. Following Chambers et al. (1996,1998), one can therefore interpret the temporal shortage function as an efficiency measure.

After this theoretical analysis of the temporal shortage function, we now turn to its estimation using a non-parametric frontier methodology.

3.3. *Non-parametric frontier estimation of the temporal shortage function*

It is well-known that technical efficiency measures can be calculated relative to non-parametric production frontiers providing piecewise linear approximations of the underlying true, but unknown technology. The resulting production boundary is simply an envelopment of observed data and any observation can be positioned relative to this boundary by computing a simple linear programming problem (e.g., Lovell, 1993). Assuming there are k DMU's over the time period, an observed technological path for any observation is evaluated using the temporal shortage function by computing the following linear program:

$$\max \frac{1}{T} \sum_{t=1}^{\mathrm{T}} \delta^t$$

$$\text{s.t. } x_n^{j_o,t} - \delta^t g_{i,n}^t \geqslant \sum_{k=1}^{K} z^{k,t} x_n^{k,t},$$
$$n = 1, \ldots, N, \quad t = 1, \ldots, T,$$

$$y_m^{j_o,t} + \delta^t g_{o,n}^t \leqslant \sum_{k=1}^{K} z^{k,t} y_m^{k,t},$$
$$m = 1, \ldots, M, \quad t = 1, \ldots, T,$$

$$\delta^t \geqslant 0, \quad z^t \in \Gamma, \quad k = 1, \ldots, K, \quad t = 1, \ldots, T,$$

where $\Gamma \in \{\Gamma^{\mathrm{CRS}}, \Gamma^{\mathrm{VRS}}, \Gamma^{\mathrm{NIRS}}, \Gamma^{\mathrm{NDRS}}\}$, with:

(i) $\Gamma^{\mathrm{CRS}} = R_+^K$,
(ii) $\Gamma^{\mathrm{VRS}} = \left\{ z \in R_+^K; \sum_{k=1}^{K} z^k = 1 \right\}$,
(iii) $\Gamma^{\mathrm{NIRS}} = \left\{ z \in R_+^K; \sum_{k=1}^{K} z^k \leqslant 1 \right\}$,

and

(iv) $\Gamma^{\mathrm{NDRS}} = \left\{ z \in R_+^K; \sum_{k=1}^{K} z^k \geqslant 1 \right\}$

representing, respectively, the following maintained returns to scale hypotheses (i) constant returns to scale; (ii) variable returns to scale; (iii) non-increasing returns to scale; and (iv) non-decreasing returns to scale.

Notice that from a computational point of view, this block-diagonal LP for each technological path can be eventually decomposed into T sub-problems, since there are no temporal linkages between each of the estimated technologies in each sub-period.

3.4. A discounted temporal shortage function

When proposing the arithmetic mean of static measures as a global technical efficiency measure, it is implicitly assumed that the time dimension is neutral. But, for an economic agent the present is more valuable than the past. To formalize this idea of positive time preference in a production context, we adapt the temporal efficiency measure by attributing most weight to the most recent efficiency measures composing it. This is accomplished by weighting the component efficiency measures by a discount factor, denoted ξ. This parameter is assumed to remain constant over time. The goal of this subsection is then to model a weighted or discounted global performance index.

Definition 4. If $\mathcal{GR}$ is a temporal production technology satisfying GR1–GR8, (X, Y) is an input–output path in $\mathcal{GR}$. $0<\xi<1$, then the discounted temporal shortage function $S^{\xi}(X, Y, G)$ is defined as follows:

$$S^{\xi}(X, Y, G) = \max_{\delta} \left\{ \sum_{t=1}^{\mathrm{T}} \frac{\xi^{T-t}\delta^t}{T} \,|\, (X, Y) + \delta G \in \mathcal{GR} \right\}.$$

This definition proposes a weighted (discounted) temporal efficiency measure, whereby the weights are lower as one moves away from the present into the past. By analogy to the temporal efficiency measure, one can immediately proof the following proposition with respect to this discounted temporal shortage function.

Proposition 3. *If $\mathcal{GR}$ is a temporal production technology satisfying GR1–GR8, (X, Y) is an input–output path in $\mathcal{GR}$,* and $\delta = (\delta^1, \delta^2, \ldots, \delta^{\mathrm{T}})$, *then the discounted temporal shortage function noted $S^{\xi}(X, Y, G)$ can be written as follows*:

$$S^{\xi}(X, Y, G) = \max_{\delta} \left\{ \sum_{t=1}^{\mathrm{T}} \frac{\xi^{T-t}\delta^t}{T} \,|\, (X, Y) + \delta G \in \mathcal{GR} \right\}$$
$$= \max_{\delta} \left\{ \sum_{t=1}^{\mathrm{T}} \frac{\xi^{T-t}\delta^t}{T} \,|\, (x^t, y^t) + \delta^t g^t \in GR^t \right\}$$

$$= \frac{1}{T} \sum_{t=1}^{\mathrm{T}} \xi^{T-t} \max_{\delta^t} \{\delta^t | (x^t, y^t) + \delta^t g^t \in GR^t\}$$

$$= \frac{1}{T} \sum_{t=1}^{\mathrm{T}} \xi^{T-t} S(x^t, y^t, g^t)$$

Proof. It is straightforward by Lemma 1. □

Thus, by analogy with the temporal shortage function, the discounted temporal shortage function corresponds to the average of discounted static shortage functions. It is straightforward to show that the properties of the temporal shortage function carry over to the discounted temporal shortage function. For reasons of space we refrain from summarizing the main properties of this discounted temporal shortage function in a proposition entirely analogous to Proposition 2.

4. Duality between temporal profit and temporal shortage functions

In this section, the main focus is on establishing a duality result between the temporal shortage function and the temporal profit function. Obviously, temporal economic objective functions are not new in the economic literature. For instance, dynamic cost functions are discussed in Sengupta (2003), while dynamic revenue and short-run profit functions are treated in Färe and Grosskopf (1996). However, we are unaware of any duality results in this type of literature. Therefore, using the temporal shortage function, compatible with the most general behavioral assumption of profit maximization, to establish a duality result may well come timely. Specialized duality results between an input-oriented (output-oriented) temporal shortage function and a temporal cost (revenue) function follow suit.

The first subsection defines the temporal profit function and studies its properties. The next subsection first formulates the main duality result. Thereafter, it looks at the definition of shadow price paths and it proposes a temporal version of the overall efficiency decomposition into temporal allocative and temporal technical components.

4.1. Temporal profit function

The profit of a firm is described by the profit function $\pi(w, p) = py - wx$. By analogy, one can define the temporal profit function of a production unit by: $PY - WX = \sum_{t=1}^{\mathrm{T}} p^t y^t - w^t x^t$, or in a more formal way by: $\prod(W, P) = \sum_{t=1}^{\mathrm{T}} \pi(w^t, p^t)$. Assuming the economic objective of the firm is to maximize its profits, one derives the following proposition:

Proposition 4. *Let $\mathcal{GR}$ be a temporal production technology satisfying $GR1$–$GR8$ and (X, Y) an input–output path in $\mathcal{GR}$. Let $(W, P) \in (R_+^{N+M})^{\mathrm{T}}$ be the price path corresponding to this input–output path. Then, the temporal profit function is*

$$\prod(W, P) = \sup_{(X,Y)\geqslant 0} \{PY - WX | (X, Y) \in \mathcal{GR}\}$$
$$= \sum_{t=1}^{\mathrm{T}} \sup_{(x^t,y^t)\geqslant 0} \{p^t y^t - w^t x^t | (x^t, y^t) \in GR^t\}.$$

Proof. We have

$$\prod(W, P) = \sup_{(X,Y)\geqslant 0} \{PY - WX | (X, Y) \in \mathcal{GR}\}$$
$$= \sup_{(X,Y)} \left\{ \sum_{t=1}^{\mathrm{T}} p^t y^t - w^t x^t \right.$$
$$\left. | (x^1, y^1, \ldots, x^{\mathrm{T}}, y^{\mathrm{T}}) \in \mathcal{GR} \right\}.$$

From Lemma 1, one derives that:

$$\prod(W, P) = \sum_{t=1}^{\mathrm{T}} \sup_{x^t,y^t} \{p^t y^t - w^t x^t | (x^t, y^t) \in GR^t\}$$
$$= \sum_{t=1}^{\mathrm{T}} \pi(w^t, p^t). \qquad \square$$

Thus, the temporal profit function corresponds to the sum of the static profit functions defined over each time period. This result is somewhat similar to aggregation results over production units developed in the literature (see Färe and Grosskopf (2004) for a survey).

Proposition 5. *When $\mathcal{GR}$ satisfies the axioms $GR1$–$GR8$, then the temporal profit function*

$\prod(W,P):(R_+^{N+M})^{\mathrm{T}} \to R_+$ satisfies the following properties:

(1) $\prod(\lambda W, \lambda P) = \lambda \prod(W,P)$.
(2) $\forall X' \geqslant X$ *and* $Y' \leqslant Y, \prod(W',P') \geqslant \prod(W,P)$.
(3) $\forall P' \geqslant P$ *and* $W' \leqslant W, \prod(W',P') \geqslant \prod(W,P)$.
(4) $\prod(W,P)$ *is continuous in* (W,P).
(5) $\prod(W,P)$ *is convex in* (W,P).

Proof. (1) Let $\lambda^t = \lambda \quad \forall t$,

$$\begin{aligned}\prod(\lambda W, \lambda P) &= \frac{1}{T}\sum_{t=1}^{\mathrm{T}} \pi(\lambda w^t, \lambda p^t) = \frac{1}{T}\sum_{t=1}^{\mathrm{T}} \lambda\pi(w^t,p^t) \\ &= \lambda\left(\frac{1}{T}\sum_{t=1}^{\mathrm{T}} \pi(w^t,p^t)\right) = \lambda\prod(W,P).\end{aligned}$$

(2) This follows from the definition of the profit function.

(3) According to Varian (1992), $\pi(w^t,p^t)$ is continuous $\forall t$ following the maximum theorem. It follows that $1/T\sum_{t=1}^{\mathrm{T}}\pi(w^t,p^t)$ is continuous.

(4) Let two price paths $(W,P),(W',P') \in (R_+^{N+M})^{\mathrm{T}}$. According to Varian (1992), we have

$$\begin{aligned}&\Rightarrow \pi(\theta w^t + (1-\theta)w'^t, \theta p^t + (1-\theta)p'^t) \\ &\quad \geqslant \theta\pi(w^t,p^t) + (1-\theta)\pi(w'^t,p'^t)\end{aligned}$$

$$\begin{aligned}&\Rightarrow \frac{1}{T}\sum_{t=1}^{\mathrm{T}} \pi(\theta w^t + (1-\theta)w'^t, \theta p^t + (1-\theta)p'^t) \\ &\quad \geqslant \frac{1}{T}\sum_{t=1}^{\mathrm{T}} \theta\pi(w^t,p^t) + (1-\theta)\frac{1}{T} \\ &\quad\quad \times \sum_{t=1}^{\mathrm{T}}(1-\theta)\pi(w'^t,p'^t)\end{aligned}$$

$$\begin{aligned}&\Rightarrow \prod(\theta W + (1-\theta)W', \theta P + (1-\theta)P') \\ &\quad \geqslant \theta\prod(W,P) + (1-\theta)\prod(W',P'). \qquad \square\end{aligned}$$

Property 1 states that the temporal profit function is homogeneous of degree 1, i.e., it varies proportionally to the price path. Property 2 implies that it is non-decreasing in relation to the output path and non-increasing in relation to the input path. The same also applies in terms of the price paths. Finally, the temporal profit function is continuous and convex with respect to the price path (W,P).

4.2. Duality relation between the temporal profit and shortage functions

We first introduce some notations that are needed in the remainder of this subsection. Let $G = (g^1,\ldots,g^{\mathrm{T}})$ be a directional path and $G \in (R_+^{N+M})^{\mathrm{T}}$. Moreover, we have $g^t = (-g_i^t, g_o^t)$ for all $t = 1,\ldots,T$. Let $(W,P) \in (R_+^{M+N})^{\mathrm{T}}$ be the price path corresponding to the output–input path. We define the product $(W,P)\times G$ as follows:

$$(W,P)\times G = \begin{pmatrix} p^1 g_o^1 + w^1 g_i^1 \\ \vdots \\ p^t g_o^t + w^t g_i^t \\ \vdots \\ p^{\mathrm{T}} g_o^{\mathrm{T}} + w^{\mathrm{T}} g_i^{\mathrm{T}} \end{pmatrix}.$$

Denote $1^{\mathrm{T}} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in R_+^{\mathrm{T}}$ the temporal unit vector.

In the same way, one can define a temporal scalar $\delta = \begin{pmatrix} \delta^1 \\ \vdots \\ \delta^{\mathrm{T}} \end{pmatrix}$.

Proposition 6. *Let $\mathcal{GR}$ be a temporal production technology and $(X,Y) \in \mathcal{GR}$ an input–output path of $\mathcal{GR}$. Let $G \in \times_{t=1}^{\mathrm{T}}(R_+^{N+M})$ be a directional path. Then, we have*

(1) $$\prod(W,P) = \sup_{(X,Y)\geqslant 0}\{PY - WX + T\cdot S(X,Y,G)\cdot(W,P)\times G\}.$$

(2) $$S(X,Y,G) = \frac{1}{T}\min_{(W,P)\geqslant 0}\left\{\prod(W,P) - (PY-WX) | (W,P)\times G = 1^{\mathrm{T}}\right\}.$$

Proof.

$$(1)\ \prod(W,P) = \sup_{(X,Y)\geqslant 0}\{PY - WX | (X,Y) \in \mathcal{GR}\}$$
$$= \sum_{t=1}^{\mathrm{T}} \pi(w^t, p^t)$$
$$= \sum_{t=1}^{\mathrm{T}} \sup_{(x^t,y^t)\geqslant 0}\{p^t y^t - w^t x^t + S(x^t, y^t, g^t) \cdot (p^t g_o^t + w^t g_i^t)\}.$$

According to Lemma 1, this yields:

$$= \sup_{\begin{pmatrix} x^1,y^1 \\ \vdots \\ x^{\mathrm{T}},y^{\mathrm{T}} \end{pmatrix}} \left\{ \sum_{t=1}^{\mathrm{T}}(p^t y^t - w^t x^t) + \sum_{t=1}^{\mathrm{T}}(S(x^t, y,^t g^t) \cdot(p^t g_o^t + w^t g_i^t)) \right\}.$$

Since $S(X,Y,G) = 1/T\sum_{t=1}^{\mathrm{T}} S(x^t, y^t, g^t)$, is straightforward to obtain:

$$= \sup_{(X,Y)\geqslant 0}\{PY - WX + T \cdot S(X,Y,G) \cdot (W,P) \times G\}.$$

$$(2)\ S(X,Y,G) = \frac{1}{T}\sum_{t=1}^{\mathrm{T}} S(x^t, y^t, g^t)$$
$$= \frac{1}{T}\sum_{t=1}^{\mathrm{T}} \sup_{\delta^t}\{\delta^t \in R_+ | (x^t, y^t) - \delta^t g^t \in GR^t\}$$
$$= \frac{1}{T}\sum_{t=1}^{\mathrm{T}} \min_{(w^t,p^t)}\{\pi(w^t,p^t) - (p^t y^t - w^t x^t) \times | p^t g_o^t + w^t g_i^t = 1\}.$$

According to Lemma 1, one obtains:

$$= \frac{1}{T} \min_{\begin{pmatrix} w^1,p^1 \\ \vdots \\ w^{\mathrm{T}},p^{\mathrm{T}} \end{pmatrix}} \left\{ \sum_{t=1}^{\mathrm{T}}(\pi(w^t,p^t) - (p^t y^t - w^t x^t)) | p^t g_o^t + w^t g_i^t = 1,\ \forall t = 1, \ldots, T \right\}$$

$$= \frac{1}{T}\min_{(W,P)}\left\{\prod(W,P) - (PY - WX) | (W,P) \times G = 1^{\mathrm{T}}\right\}. \qquad \square$$

The duality between the temporal profit and shortage functions can be summarized as follows. The first part of the proposition establishes that the temporal profit function corresponds to the maximum of the observed temporal profit increased by the temporal shortage function normalized over the time horizon. The second part indicates that the temporal shortage function corresponds to the average of the difference between the temporal profit function and the observed temporal profits.

Fig. 3 illustrates the above Proposition 6 when $n = m = 1$. Along the time axis, one observes for each technology prevailing in a given time period how an eventually inefficient input–output path is projected onto the boundary of technology and how a profit hyperplane supports the same projection point for a specific price path. Both the observed and optimal technological paths are traced.

Starting from the temporal profit function and the temporal shortage function, it is straightforward to find a shadow price path. Recall that the temporal shortage function provides a complete primal representation of the temporal technology. Moreover, thanks to the envelope theorem, duality theory makes it possible to find the shadow prices supporting the frontier projections of each observed input–output path. Thus, the temporal shortage function allows deriving a shadow price path. This makes the temporal shortage function a powerful tool, similar to the traditional production function, especially in the dual price space because of its connection to the temporal profit function.

Definition 5. Let $\mathcal{GR}$ be a temporal production technology satisfying GR1–GR8. The point to set correspondence $(\tilde{W}, \tilde{P}) : \mathcal{GR} \to 2^{(R_+^{N+P})^{\mathrm{T}}}$ defined as

$$(\tilde{W},\tilde{P})(X,Y) = \arg\min_{(W,P)\geqslant 0}\left\{\prod(W,P) - (PY - WX) | (W,P) \times G = 1^{\mathrm{T}}\right\}$$

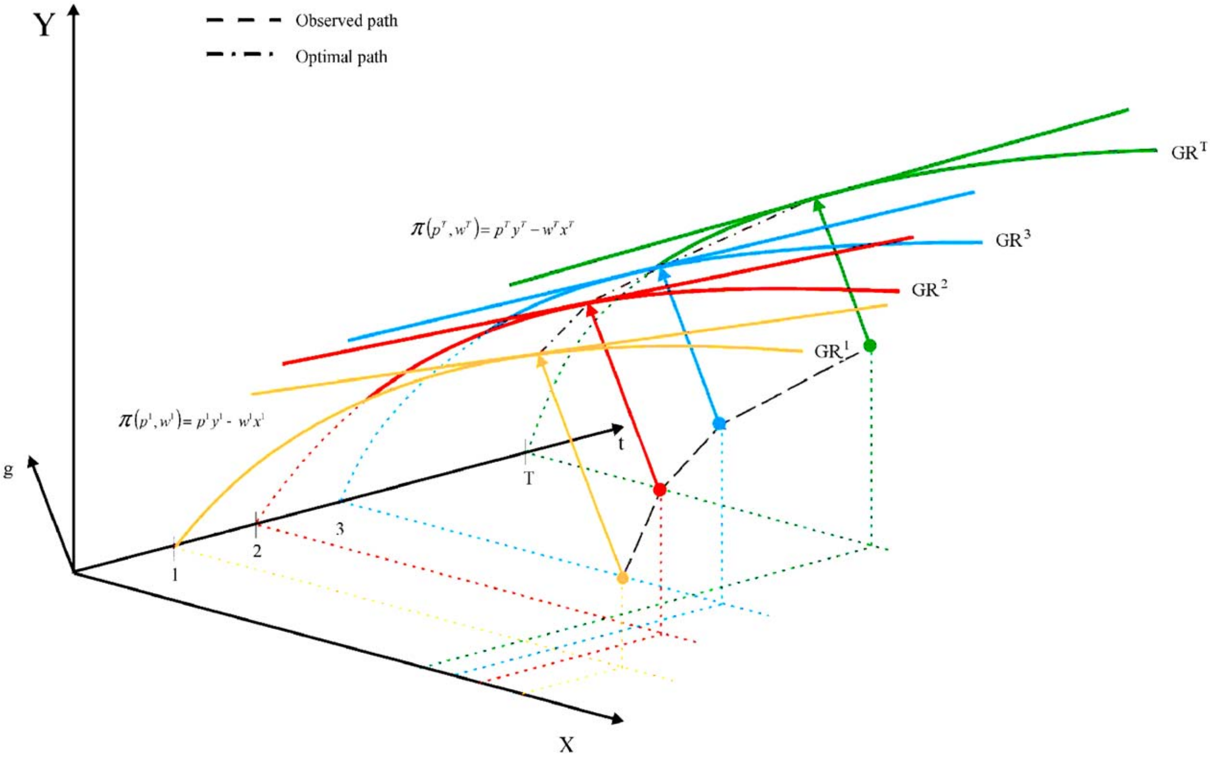

Fig. 3. Duality between temporal shortage and profit functions.

is called the temporal adjusted price correspondence.

Following Definition 7, the temporal adjusted price correspondence establishes a link between an observed input–output path and the shadow price paths minimizing the average of the difference between the temporal profit function and the observed temporal profits (see Proposition 6). Along this line—assuming differentiability of the temporal shortage function—the following result is established.

Proposition 7. *Let $\mathcal{GR}$ be a temporal production technology satisfying GR1–GR8 and (X, Y) an input–output path in $\mathcal{GR}$. For the entire path, whenever $(\tilde{W}, \tilde{P})$ is single-valued, then the temporal shortage function is differentiable and we obtain*:

$$\left(\frac{\partial S(x^t, y^t, g^t)}{\partial x^t}; \frac{\partial S(x^t, y^t, g^t)}{\partial y^t}\right)_{t=1}^{\mathrm{T}} = (\tilde{W}, -\tilde{P}).$$

Proof. If the temporal adjusted price correspondence is single valued, then the temporal shortage function is differentiable. Then, the result is a direct consequence of the envelope theorem, which is obtained by differentiating the temporal adjusted price correspondence. □

This proposition indicates that—under some regularity conditions—the total derivative of the temporal shortage function allows finding the price path solution for the maximization of the temporal profit function, i.e., the shadow price path. Notice that the above proposition guarantees uniqueness of the obtained shadow prices. An alternative way to obtain unique shadow prices is to impose a strict version of convexity on the temporal production technology (i.e., assuming a strict version of GR8). But this would exclude, for instance, imposing the hypothesis of constant returns to scale on the temporal technology.

However, the above approach imposes slightly milder assumptions and is therefore to be preferred.

A direct application of duality is the definition and decomposition of overall efficiency. Similar to the proposition in Farrell (1957), overall efficiency can be separated into technical and allocative efficiency. To see this, let us take up again Proposition 6:

$$\prod(W,P) = \sup_{(X,Y)\geqslant 0} \{PY - WX + T \cdot S(X,Y,G) \cdot (W,P) \times G\}.$$

Noticing that the temporal profit function is given for the maximum of the temporal profit function, one can write:

$$\prod(W,P) \geqslant PY - WX + T \cdot S(X,Y,G) \cdot (W,P) \times G.$$

After some rearranging, one obtains

$$\frac{1}{T}\frac{\prod(W,P) - (PY - WX)}{(W,P)\times G} \geqslant S(X,Y,G)\cdot$$

The term on the left-hand side corresponds to the measure of the temporal overall efficiency, denoted $SOE(W,P,X,Y)$. The term on the right-hand side corresponds to the temporal technical efficiency, denoted $STE(X,Y)$. Notice that $STE(X,Y) = S(X,Y,G)$. Finally, temporal allocative efficiency $SAE(W,P,X,Y)$ is defined as the difference between these two efficiency components:

$$SAE(W,P,X,Y) = SOE(W,P,X,Y) - STE(X,Y).$$

Finally, the decomposition of temporal profit efficiency can be summarized as follows:

$$SOE(W,P,X,Y) = STE(X,Y) + SAE(W,P,X,Y).$$

Remark that in line with Section 3.4 it is possible to define a discounted temporal profit function where profits in the distant past receive less weight than those close to the present. Then, all properties and duality results developed in this section could be duplicated without any difficulty. Furthermore, it is also possible to separate out another type of technical inefficiency known as congestion. This would simply require evaluating temporal technical efficiency relative to both weakly (GR3 and GR6) and strongly (GR4 and GR7) disposable technologies (see Färe et al. (1985) for this development using traditional radial efficiency measures that yield a multiplicative rather than an additive decomposition).

5. Aggregation of production over time

Recently, there has been an active interest in investigating the conditions under which firm performance indicators can be aggregated across firms to evaluate the performance of an industry (see Färe and Grosskopf (2004) for a recent survey of these issues). In a similar vein, we ask here whether it is possible to aggregate the performance of a firm over time: how does the performance of the firm average over time relate to the average performance of the firm within a given time period. The performance of a firm average over time is somewhat related to the structural efficiency notion. The latter notion is essentially an efficiency index over an entire industry allowing for reallocation of inputs and outputs among the firms composing the industry. In the case of the performance of a firm average over time, one allows for reallocations of production over time within each firm.

First, we specify more precisely what we mean by an efficiency index satisfying a temporal aggregation condition.

Definition 6. Let $\mathcal{GR}$ be a temporal production technology and $(X,Y) \in \mathcal{GR}$ an input–output path of $\mathcal{GR}$. Assume that $G = \times_{t=1}^{\mathrm{T}} g = g^{\mathrm{T}}$ where $g \in R_+^{M+N}$. Let us consider the aggregate shortage function defined by

$$AS\left(\frac{1}{T}\sum_{t=1\ldots T}(x^t,y^t);g\right) = \sup\left\{\delta : \frac{1}{T}\sum_{t=1\ldots T}(x^t,y^t) + \delta g \in \frac{1}{T}\sum_{t=1\ldots T} GR^t\right\}.$$

We say that $(X, Y) \in \mathcal{GR}$ satisfies the temporal aggregation condition if

$$AS\left(\frac{1}{T}\sum_{t=1\ldots T}(x^t, y^t), g\right) = S(X, Y, g^{\mathrm{T}}).$$

In words, the temporal aggregation condition is satisfied when the aggregate shortage function (evaluating the performance of the firm average over time) equals the temporal shortage function. As the following proposition indicates, it turns out that this condition ensuring consistent aggregation over time is rather strong.

Proposition 8. *Let $\mathcal{GR}$ be a temporal production technology and $(X, Y) \in \mathcal{GR}$ an input–output path of $\mathcal{GR}$. We have*:

$$AS\left(\frac{1}{T}\sum_{t=1\ldots T}(x^t, y^t), g\right) \geqslant S(X, Y, g^{\mathrm{T}}).$$

Proof. By definition, we have the following relationship:

$$\frac{1}{T}\sum_{t=1\ldots T}(x^t, y^t) + \frac{1}{T}\sum_{t=1\ldots T} S(x^t, y^t, g)\cdot g \in \frac{1}{T}\sum_{t=1\ldots T} GR^t.$$

Therefore, we obtain the inequality

$$AS\left(\frac{1}{T}\sum_{t=1\ldots T}(x^t, y^t), g\right) \geqslant \frac{1}{T}\sum_{t=1\ldots T} S(x^t, y^t, g) = S(X, Y, g^{\mathrm{T}}).$$

This terminates the proof. □

Indeed, the aggregate shortage function is larger or equal to the temporal shortage function. This result is similar to one obtained for the aggregation over firms (see Färe et al. (2001), Briec et al. (2003)).

Following Briec et al. (2003), this inequality allows defining a measure of aggregation bias over time between both performance measures.

Definition 7. Let $\mathcal{GR}$ be a temporal production technology and $(X, Y) \in \mathcal{GR}$ an input–output path of $\mathcal{GR}$. The difference:

$$TAB(X, Y, g^{\mathrm{T}}) = AS\left(\frac{1}{T}\sum_{t=1\ldots T}(x^t, y^t), g\right) - S\left(X, Y, g^{\mathrm{T}}\right)$$

is called the temporal aggregation bias.

Obviously, we note that $TAB(X, Y, g^{\mathrm{T}}) \geqslant 0$ for all $(X, Y) \in \mathcal{GR}$. Other properties have been developed in Briec et al. (2003) and could be similarly derived.

Having dealt with technical efficiency, we turn our attention to the effect of aggregation over time on the measures of overall and allocative efficiency. First, we define an index of structural overall efficiency as follows:

$$SOE(W, P, X, Y) = \frac{1}{T}\sum_{t=1\ldots T}\frac{\pi(w^t, p^t) - (p^t y^t - w^t x^t)}{(w^t, p^t).g} = \frac{1}{T}\sum_{t=1\ldots T} OE(w^t, p^t, x^t, y^t),$$

where

$$OE(w^t, p^t, x^t, y^t) = \frac{\pi(w^t, p^t) - (p^t y^t - w^t x^t)}{(w^t, p^t).g}.$$

In other words, structural overall efficiency equals the time average of the static firm overall efficiencies. This identity is similar to the Koopmans (1957) result about the aggregation of profit functions over firms within an industry. Now we define the aggregate overall efficiency as the performance of the firm average over time by:

$$AOE\left(\sum_{t=1\ldots T}(x^t, y^t), W, P, g\right) = \frac{1}{T}\sup\left\{\delta : \sum_{t=1\ldots T}(-w^t, p^t)[(x^t, y^t) - \delta g] \leqslant \sum_{t=1\ldots T}\pi^t(w^t, p^t)\right\}.$$

Following these developments above, we derive the identity:

$$AOE\left(\sum_{t=1\ldots T}(x^t, y^t), W, P, g\right) = SOE(W, P, X, Y) = \frac{1}{T}\sum_{t=1\ldots T} OE(w^t, p^t, x^t, y^t).$$

Hence, aggregate overall efficiency equals structural overall efficiency, a result similar to the one in Briec et al. (2003) on the aggregation across firms.

Finally turning to the allocative efficiency component, we introduce two more concepts. First, the aggregate allocative efficiency is defined by:

$$AAE(X, Y, W, P, g^{\mathrm{T}}) = AOE\left(\sum_{t=1\ldots T}(x^t, y^t), W, P, g\right) - AS\left(\frac{1}{T}\sum_{t=1\ldots T}(x^t, y^t), g\right).$$

Second, we define the structural allocative efficiency as follows:

$$SAE(W, P, X, Y) = SOE(W, P, X, Y) - S(X, Y, g^{\mathrm{T}}).$$

Now we are in a position to connect both the aggregate allocative efficiency and the structural allocative efficiency notions to the temporal aggregation bias introduced in Definition 7.

Proposition 9. *Let $\mathcal{GR}$ be a temporal production technology and $(X, Y) \in \mathcal{GR}$ an input–output path of $\mathcal{GR}$. We have*

$$SAE(W, P, X, Y) - AAE(X, Y, W, P, g^{\mathrm{T}}) = TAB(X, Y, g^{\mathrm{T}}).$$

Proof. From Koopmans (1957), we have shown that $SOE(W, P, X, Y) = AOE(W, P, X, Y)$. Consequently:

$$\begin{aligned} SOE(W, P, X, Y) &= SAE(W, P, X, Y) + \frac{1}{T}\sum_{t=1}^{\mathrm{T}} S(x^t, y^t, g) \\ &= SAE(W, P, X, Y) + S(X, Y, g^{\mathrm{T}}) \\ &= AAE(X, Y, W, P, g^{\mathrm{T}}) + AS\left(\sum_{t=1\ldots T}(x^t, y^t); g\right). \end{aligned}$$

Therefore, we deduce that

$$\begin{aligned} SAE(W, P, X, Y) - AAE(X, Y, W, P, g^{\mathrm{T}}) &= AS\left(\sum_{t=1\ldots T}(x^t, y^t); g\right) - S(X, Y, g^{\mathrm{T}}) \\ &= TAB(X, Y, g^{\mathrm{T}}). \qquad \square \end{aligned}$$

Thus, structural allocative efficiency is larger or equal to aggregate allocative efficiency and the temporal aggregation bias (being positive) fills up the gap between both. This result is similar to Corollary 1 in Briec et al. (2003).

Proposition 10. *Let $\mathcal{GR}$ be a temporal production technology and $(X, Y) \in \mathcal{GR}$ an input–output path of $\mathcal{GR}$. We have*:

$$SAE(W, P, X, Y) \geqslant TAB(X, Y, g^{\mathrm{T}}).$$

Proof. We have shown that

$$SAE(W, P, X, Y) - AAE(X, Y, W, P, g^{\mathrm{T}}) = TAB(X, Y, g^{\mathrm{T}}).$$

But, $AAE(X, Y, W, P, g^{\mathrm{T}}) \geqslant 0$. Consequently, $SAE(X, Y, W, P, g^{\mathrm{T}}) \geqslant TAB(X, Y, g^{\mathrm{T}})$. $\square$

Thus, structural allocative efficiency is larger or equal to the temporal aggregation bias. The temporal aggregation bias thus provides a lower bound for the structural allocative efficiency measure. This last result duplicates exactly the aggregation results over firms developed in Proposition 8 of Briec et al. (2003).

6. Conclusions

This paper has offered a temporal generalization of the popular analysis of static efficiency measurement. The temporal efficiency measure generalizes the shortage function, introduced by Luenberger (1992, 1995) and Chambers et al. (1996, 1998). The definition of temporal technical efficiency allows us to verify the efficiency in panel data of production units, while ignoring the possibility of technological change and its precise measurement. Moreover, the development of a temporal duality result between the temporal shortage and profit functions allows obtaining a shadow price path and a temporal inefficiency decomposition. Finally, some aggregation results were derived allowing some statements about the average performance of a unit over time.

Obvious potential extensions of this approach are the derivation of similar temporal analysis for the special cases of the (i) input-oriented directional distance function and the cost function and the (ii) output-oriented directional distance function and the revenue function. Equally so, the derivation of the detailed results for the discounted temporal shortage function may be worthwhile pursuing. In addition, it could be valuable to extend our development by linking it to the literature allowing for time substitution (e.g., Färe and Grosskopf, 1996; Sengupta, 1995, 2003) or for dynamic phenomena like adjustment costs (e.g., Sengupta, 1992, 1999).

We hope this contribution proves inspiring when evaluating the performance of industries where technological change is a priori of little relevance because of its embodied nature in large and indivisible infrastructures.

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