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The Malmquist Productivity Index and Plant Capacity Utilization*

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Abstract

A new decomposition of the Malmquist productivity index is proposed to account for changes in plant capacity utilization. Using a primal, non-parametric specification of technology, the Malmquist index is decomposed into technical efficiency change, variations in plant capacity utilization and frontier shifts. It provides an alternative to the available methods of incorporating capacity utilization changes into measures of productivity change. Such measures are based on parametric (and, in many cases, dual) technology specifications; moreover, they typically do not allow for technical inefficiency.

Keywords: Productivity; capacity utilization

JEL classification: *C*61; *D*24

I. Introduction

The Malmquist productivity index, defined by Caves *et al.* (1982) as a ratio of distance functions, has gained some popularity in applied work; see e.g. Berg *et al.* (1992). More recently, Färe *et al.* (1995) developed a straightforward computational procedure to calculate the Malmquist index relative to non-parametric frontier technologies by exploiting the inverse relationship between output distance functions and output-oriented technical efficiency measures.[1] Furthermore, they relaxed the implicit hypothesis of technical efficiency maintained in Caves *et al.* (1982) and showed that the Malmquist productivity index can be decomposed into technical efficiency changes and technology shifts.

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[1] Given the difficulties involved in computing distance functions at the time, Caves *et al.* (1982) approximated the Malmquist productivity index by a Törnqvist index. The latter is computable from price and quantity data only, and does not require a precise knowledge of technology. Unfortunately, the conditions under which it provides a good approximation to the Malmquist index are quite stringent. The Färe *et al.* (1995) methodology makes recourse to the Törnquist index unnecessary.

One potentially important issue ignored in applications of the Malmquist productivity index is that changes in technical efficiency may be partially due to changes in the utilization of production capacity. The purpose of this paper is therefore to suggest a further decomposition of the Malmquist productivity index to separate technical efficiency changes from variations in capacity utilization. More specifically, we integrate Johansen's (1968) measure of plant capacity utilization into the Malmquist index; see also Färe *et al.* (1989a). This allows us to decompose productivity changes into frontier shifts, variations in technical efficiency and variations in capacity utilization.

Integrating variations in plant capacity utilization in the development of the Malmquist productivity index is a welcome addition to the literature for several reasons. First, for parametric specifications of technology, various productivity decompositions have been suggested to incorporate measures of capacity utilization; see, among others, Hulten (1986) and Morrison (1985, 1993). However, such decompositions are as yet unavailable for non-parametric technology representations. Second, unlike the available parametric approaches, the decomposition suggested here incorporates the possibility of technical inefficiency. Third, since the Malmquist index has so far been defined for primal technologies only, we use a primal concept of capacity, unlike previous studies based on dual representations of technology such as Morrison (1985) and Squires (1987). A primal approach is a useful alternative that may be especially relevant in situations where prices are unreliable or unavailable.

The structure of the paper is as follows. In Section II we begin by reviewing the Malmquist productivity index and the Johansen (1968) measure of plant capacity utilization. We then show how Malmquist productivity changes can be decomposed into frontier shifts, variations in technical efficiency and variations in capacity utilization. Section III concludes.

II. Malmquist Productivity Indices and Capacity Utilization

Malmquist Productivity Indices

Assume that for periods $t = 1, \ldots, T$ we observe m inputs ($x^t \in \Re^m_+$) producing n outputs ($y^t \in \Re^n_+$). In each period t, the production technology is defined by the set of feasible input/output vectors: $S^t = \{(x^t, y^t) | x^t \text{ can produce } y^t\}$. The output set $P^t(x^t)$ denotes all output vectors y^t that can be produced from the input vector x^t, i.e., $P^t(x^t) = \{y^t | (x^t, y^t) \in S^t\}$. The output distance function is defined as:

$$D^t_o(x^t, y^t) = \min\{\theta | (y^t/\theta) \in P^t(x^t)\}. \quad (1)$$

It treats the inputs as given and expands, in a proportional way, the output vector until $y^t/D_o^t(x^t, y^t)$ belongs to the isoquant of the output set (Isoq $P^t(x^t)$). For the subsequent analysis, it is important to note that output distance functions are inversely related to the radial technical efficiency measures in the outputs; see Färe *et al.*, (1994). Denoting radial output efficiency by $DF_o^t(x^t, y^t)$ we have

$$DF_o^t(x^t, y^t) = 1/D_o^t(x^t, y^t). \tag{2}$$

An output-based Malmquist productivity index with base period t was defined by Caves *et al.* (1982) as the ratio of two output distance functions:

$$M_o^t(x^t, y^t, x^{t+1}, y^{t+1}) = D_o^t(x^{t+1}, y^{t+1})/D_o^t(x^t, y^t), \tag{3}$$

where $D_o^t(x^t, y^t)$ and $D_o^t(x^{t+1}, y^{t+1})$ are output distance functions relating observations in period t and $t+1$, respectively, to a period t technology. Of course, a Malmquist productivity index in the outputs with base period $t+1$ can similarly be defined as

$$M_o^{t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = D_o^{t+1}(x^{t+1}, y^{t+1})/D_o^{t+1}(x^t, y^t). \tag{4}$$

Therefore, to avoid an arbitrary choice of base period, Färe *et al.* (1995) proposed defining the output-oriented Malmquist productivity index as a geometric mean of (3) and (4):

$$\begin{aligned} &M_o^{t,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) \\ &= \sqrt{M_o^t(x^t, y^t, x^{t+1}, y^{t+1}) \cdot M_o^{t+1}(x^t, y^t, x^{t+1}, y^{t+1})} \\ &= \sqrt{\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \cdot \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^t, y^t)}}. \end{aligned} \tag{5}$$

The base period of this productivity index changes over time. It can be conceptualized as an index computed in a two-year window sliding over the observations in time. Moreover, the Malmquist index (5) can be decomposed into two mutually exclusive components:

$$\begin{aligned} &M_o^{t,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) \\ &= \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \sqrt{\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1})} \cdot \frac{D_o^t(x^t, y^t)}{D_o^{t+1}(x^t, y^t)}}. \end{aligned} \tag{6}$$

The first component measures the change in technical efficiency over time, while the second is related to the shifts of the frontier of the production technology, i.e., it captures technical change. If $M_o^{t,t+1}(x^t, y^t, x^{t+1}, y^{t+1})$ is larger (smaller) than unity, this indicates an improvement (deterioration) in productivity.[2] A similar interpretation applies to the separate components.

Capacity Utilization

So as to maintain consistency with the primal approach underlying the Malmquist productivity index, this paper focuses on a primal definition of capacity. In particular, we follow the seminal contribution by Johansen (1968) who defined plant capacity as the maximal amount that can be produced per unit of time with existing plant and equipment without restrictions on the availability of variable production factors. It is clear that this notion of capacity is a technical (engineering) concept that, unlike economic (cost) capacity notions, is not based on optimizing behaviour.[3] Of course, this can be considered a disadvantage from an economic point of view. However, such drawbacks should be traded off against various more attractive features of Johansen's concept. For example, Johansen (1968) shows that, almost independent of the production model assumed (fixed proportions, neoclassical production, or putty-clay models), his capacity concept meets many of the criteria typically required (existence, attainability, aggregation, consistency, etc.). He also argues that most estimation methods, such as engineering and survey-based methods, explicitly or implicitly have this specific plant capacity concept in mind.[4] Moreover, avoiding any strong hypotheses on the optimizing behaviour of the organizations under scruntiny may be a clear advantage in cases where the hypotheses are unlikely to hold (e.g., in the public sector). Rather than embarking on a full discussion of the relative merits of different capacity notions, we simply note that the relations between several economic and technical capacity measures are well known; see Nelson (1989).

[2] The technical efficiency change component has been further decomposed into variations in technical efficiency, scale efficiency and congestion; see Färe *et al.* (1994). The important issue of identifying scale effects in the technical change component has led to a discussion from which no consensus has yet emerged; see e.g. Balk (1998) and Färe *et al.* (1998).

[3] One capacity notion considers the output produced at short-run minimum average total cost, given existing plant and factor prices, as in e.g. Morrison (1985). Another definition looks at the output for which short- and long-run average total cost curves are tangent, as in e.g. Segerson and Squires (1990). Both coincide under constant returns to scale.

[4] Christiano (1981) presents an overview of both data-based and survey methods for estimating capacity utilization. He acknowledges (p. 171) that many survey respondents have an engineering concept in mind.

A method for calculating Johansen's plant capacity utilization measure using non-parametric, deterministic specifications of technology has been introduced in Färe *et al.* (1989a, 1989b). Partitioning the input vector $x^t = (x^t_f, x^t_v)$ into fixed (x^t_f) and variable (x^t_v) inputs at period *t*, plant capacity utilization $(PCU^t(x^t, x^t_f, y^t))$ at time *t* is defined as follows:

$$PCU^t_o(x^t, x^t_f, y^t) = \frac{DF^t_o(x^t, y^t)}{DF^t_o(x^t_f, y^t)}, \tag{7}$$

where $DF^t_o(x^t, y^t)$ and $DF^t_o(x^t_f, y^t)$ are output efficiency measures relative to technologies including, respectively excluding, the variable inputs. By definition $DF^t_o(x^t_f, y^t) \geqslant DF^t_o(x^t, y^t) \geqslant 1$, hence $PCU^t(x^t, x^t_f, y^t) \leqslant 1$. Any deviation from unity is interpreted as a proportional decrease in actual outputs compared to outputs at full plant capacity. Importantly, note that the plant capacity utilization index (7) is obtained by first removing any existing technical inefficiency; indeed, it is computed as a ratio of efficiency measures. Elimination of inefficiencies implies that it is not downward biased, in contrast to most traditional capacity utilization measures.

Malmquist Productivity Indices and Plant Capacity Utilization

In order to isolate changes in capacity utilization in the definition of productivity increases, we suggest the following decomposition of the Malmquist productivity index so as to explicitly separate variations in plant capacity utilization, changes in efficiency and pure technical change. We start by noting that technical efficiency can be written as

$$\begin{aligned} DF^t_o(x^t, y^t) &= DF^t_o(x^t_f, y^t) \cdot \frac{DF^t_o(x^t, y^t)}{DF^t_o(x^t_f, y^t)} \\ &= DF^t_o(x^t_f, y^t) \cdot PCU^t_o(x^t, x^t_f, y^t). \end{aligned} \tag{8}$$

The last equality is based on (7) above; see also Färe (1989a). In other words, technical efficiency equals the product of technical efficiency relative to a full capacity (short-run) technology and plant capacity utilization.

Using (8) and the relation between the radial efficiency measure and the output distance functions in (2), we can decompose the technical efficiency change component of the Malmquist productivity index

$$M^{t,t+1}_o(x^t, y^t, x^{t+1}, y^{t+1}).$$

Indeed, incorporating (2) and (8) in (6), we obtain:

$$M_o^{t,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_o^{t+1}(x_f^{t+1}, y^{t+1})}{D_o^t(x_f^t, y^t)} \cdot \frac{PCU^t(x^t, x_f^t, y^t)}{PCU^{t+1}(x^{t+1}, x_f^{t+1}, y^{t+1})} \times \sqrt{\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1})} \cdot \frac{D_o^t(x^t, y^t)}{D_o^{t+1}(x^t, y^t)}}. \quad (9)$$

Expression (9) shows that productivity changes are the combined result of three separate phenomena. The first component measures the change in technical efficiency assuming a constant degree of capacity utilization. Specifically, it evaluates the change in technical efficiency relative to a full capacity output technology between periods t and $t+1$. The second component captures the change in the degree of plant capacity utilization between t and $t+1$, holding technical efficiency levels constant. The third component is the same as in (6) and reflects pure technical change. When any of the components is larger (smaller) than unity, this indicates an improvement (deterioration) in the corresponding component, except for the component indicating changes in plant capacity utilization. For the latter, a number smaller (larger) than unity indicates an improvement (deterioration). In other words, this decomposition of the Malmquist productivity index provides a straightforward procedure for relating productivity growth to the dynamics of capacity utilization.

The decomposition of the Malmquist index suggested here is easily calculated for non-parametric technologies. Since (9) only requires data on inputs and outputs (no price information is necessary) and given the importance of correctly evaluating changes in capacity utilization, this index may have relevant empirical applicability. At the microeconomic level, for example, firm panel data could be used to separate variations in technical efficiency from variations in capacity utilization. At the macroeconomic level, capacity utilization is highly relevant as a leading indicator for inflation and business cycle movements; see Corrado and Mattey (1997). Given this relevance, an easily computable procedure for separating efficiency changes and variations in capacity utilization may be important for the correct interpretation of both macroeconomic times series and cross-national comparisons of macro data. In this respect, the index provides an interesting alternative to macroeconomic estimation methods based on surveys; see Christiano (1981).

III. Conclusion

In this paper we have integrated the measurement of plant capacity into the definition of the Malmquist productivity index. This index was decomposed into technical efficiency changes, variations in capacity utilization, and technical change. This is important since what might otherwise appear as inefficiency in a standard application of the Malmquist index may to some extent be explained by variations in capacity utilization.

We end by noting a vital avenue of research. It would be highly useful to develop discrete time dual technical change indices; see Balk (1998) for a recent proposal. These could then be similarly extended by means of economic capacity notions to capture changes in capacity utilization.

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