Radial and Nonradial Technical Efficiency Measures on a DEA Reference Technology: A Comparison Using US Banking Data

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1 Introduction

A burgeoning literature on the nonparametric measurement of technical efficiency now exists. The deterministic reference technology relative to which efficiency is measured under this approach can be constructed in a variety of ways. The empirical effect of the choice among different reference technologies on the traditional radial measures is well documented[(1)]. However, little attention has been devoted to the reverse problem, the consequences of choosing different measures of technical efficiency for a given reference technology. The first objective of this paper is to gain a better understanding of the economic interpretation and the justification of different measures of technical efficiency. We therefore succinctly review the efficiency measures that have been proposed in the axiomatic literature[(2)]. The second purpose is to offer an empirical illustration on a sample of US banks of the effects of using different efficiency measures on one of the more popular reference technologies, variable returns to scale data envelopment analysis (DEA) with strong disposability in inputs and outputs (Afriat [1972], Banker, Charnes and Cooper [1984]). An analoguous empirical assessment for the same data on a free disposal hull (FDH), which relaxes the convexity assumption (see, e.g., Tulkens [1993]), is provided in De Borger, Ferrier and Kerstens [1994].

[(1)] For example, see Grosskopf [1986] for a review of the relations between the radial efficiency measures calculated on a variety of DEA technologies.

[(2)] A detailed discussion of this axiomatic literature on efficiency measurement is found in Kerstens and Vanden Eeckaut [1994].

Most of the empirical literature on efficiency measurement is based on the radial or equiproportionate measures developed by Debreu [1951] and Farrell [1957], while the alternative nonradial measures have been largely ignored (Deller and Nelson [1991] is a noteworthy exception). Though popular, the radial measures have an important shortcoming — they fail to satisfy the intuitively appealing Koopmans [1951] definition of technical efficiency. The Koopmans definition equates technical efficiency with membership in the efficient subset, whereas the radial measures define technical efficiency relative to the isoquant. Many popular parametric forms (e.g., Cobb-Douglas) impose equality between the isoquant and the efficient subset, therefore this drawback of the radial measure may not be very important when using parametric techniques. However, for the popular reference technologies used in the nonparametric approaches (e.g., the various DEA models), this issue is potentially very important since the isoquant and the efficient subset are likely to diverge (see Färe, Grosskopf and Lovell [1985]).

The second section of this paper documents this divergence between the definition and the measurement of technical efficiency. Some pragmatic solutions to this problem are also considered, but they are found to be wanting. Section 3 provides a brief review of the theoretical literature that offers various solutions to the divergence problem. This literature debates the set of properties that an ideal measure of technical efficiency should satisfy and proposes various nonradial measures of technical efficiency that satisfy some of the properties. Because the importance of the divergence problem is largely an empirical matter (especially for DEA models), section 4 offers a systematic analysis of the impact of using alternative measures of technical efficiency based on a common reference technology (variable returns to scale DEA), an issue that has been almost completely ignored in the literature. To the best of our knowledge, this is the first empirical paper that explores the alternative efficiency measures in DEA[(3)]. Section 5 summarizes the paper's conclusions.

2 The divergence between the definition of and the radial measurement of technical efficiency

This section illustrates a major shortcoming of the radial measures of efficiency. For simplicity, the discussion concentrates on input efficiency, with the input correspondence $L(y)$ serving as the representation of technology. In defining measures of technical efficiency, three

[(3)] Deller and Nelson [1991] only report the Färe-Lovell efficiency measure, which is defined below.

subsets of the input correspondence $L(y)$ merit particular attention (see Färe, Grosskopf and Lovell [1985], Färe and Hunsaker [1986]). Assuming that output is semipositive, these subsets are the input isoquant of $L(y)$[4]:

$$\text{Isoq}\, L(y) = \Big\{x \mid x \in L(y), \lambda x \notin L(y) \quad \forall \lambda \in [0,1)\Big\};$$

the weak efficient subset of $L(y)$:

$$\text{W Eff}\, L(y) = \Big\{x \mid x \in L(y), x' < x \Rightarrow x' \notin L(y)\Big\};$$

and the efficient subset of $L(y)$:

$$\text{Eff}\, L(y) = \Big\{x \mid x \in L(y), x' \leqslant x \Rightarrow x' \notin L(y)\Big\}.$$

All three subsets denote production on the boundary of $L(y)$. The efficient subset is a (possibly proper) subset of the weak efficient subset, whereas the weak efficient subset is a (possibly proper) subset of the isoquant. That is, $\text{Isoq}\, L(y) \supseteq \text{W Eff} L(y) \supseteq \text{Eff} L(y)$.

Figure 1 shows typical isoquants for a DEA technology with strong disposability of inputs and a DEA technology with weak disposability of inputs. For both technologies, the efficient subset, $\text{Eff}\, L(y)$, consists of the line segments joining points BCDE. For the technology with strong disposability, $\text{W Eff}\, L(y)$ and $\text{Isoq}\, L(y)$ coincide[5] — both contain the connected line segments ABCDEF and the dashed lines beyond A and F parallel to the axes. For the weakly disposable technology, $\text{W Eff}\, L(y)$ contains the connected line segments ABCDEF, and $\text{Isoq}\, L(y)$ is formed by adding the line segment FG to $\text{W Eff}\, L(y)$. Points on the rays through 0A and 0G are not part of the isoquant, though they do belong to the boundary of the input correspondence. Note that $\text{W Eff}\, L(y)$ contains line segments parallel to the axes where the marginal productivity of input equals zero, and that $\text{Isoq}\, L(y)$ can be backward bending, implying negative marginal productivity or congestion.

Because both DEA technologies share a common efficient subset, the divergence between the Koopmans definition and the radial efficiency measure depends on the number of observations outside the cone

[4] Vector inequality conventions in the text: $x \geqq y$ if and only if $x_i \geqslant y_i$ for all i; $x \geqslant y$ if and only if $x_i \geqslant y_i$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ for all i; and $x > y$ if and only if $x_i > y_i$ or $x_i = y_i = 0$ for all i.

[5] Strong disposability of the inputs is a sufficient condition for $\text{Isoq}\, L(y)$ and $\text{W Eff}\, L(y)$ to coincide. See Färe, Grosskopf and Lovell ([1985] pp. 31–32) for details.

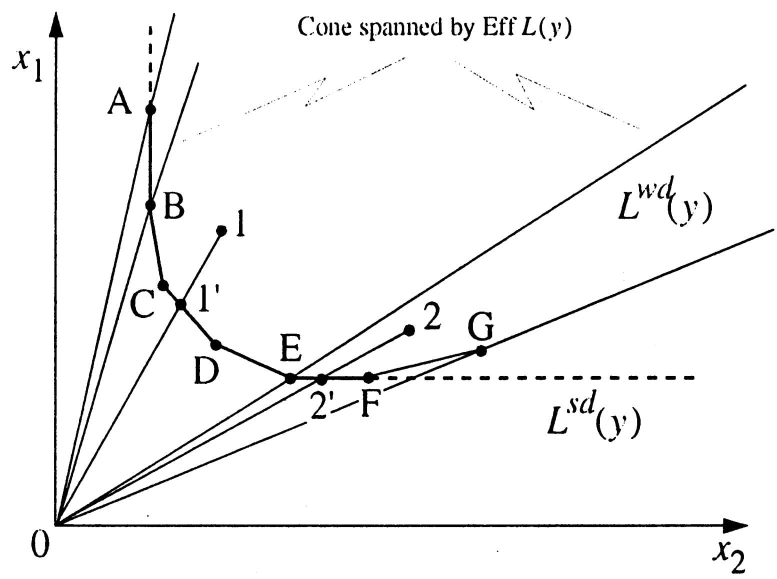

Figure 1: Three subsets of the input correspondence for DEA models.

spanned by the efficient subset (this cone is spanned by the rays 0B and 0E in Figure 1). This number is an empirical issue, but so far it has received little attention in the literature.

The crucial point is that two different notions of technical efficiency have emerged in the economics literature. The first, due to Debreu [1951] and Farrell [1957], is a radial notion defining (input-based) technical efficiency as one minus the maximum equiproportionate reduction in all inputs that still allows production of the observed level of outputs. That is, it is the ratio of the smallest feasible contraction of an observed input vector to the observed input vector itself. Formally, the Debreu-Farrell input-based measure of technical efficiency is given by:

$$DF_i(x, y) = \min \{\lambda \mid \lambda \geqslant 0,\ \lambda x \in L(y)\}.$$

DF_i varies between zero and one, with unity representing efficient production. This measure adopts the isoquant as its reference for defining technical efficiency. Therefore, an observation is efficient under a radial measure if and only if it belongs to the isoquant; it is inefficient otherwise.

The second notion of efficiency is due to Koopmans [1951], who provided a definition of technical efficiency without proposing how to measure it. In his now classic definition, a producer is deemed technically efficient if an increase in any output requires a decrease in at least one of the other outputs, or if a decrease in any input requires an increase in at least one of the other inputs. This definition of technical efficiency clearly centers on the efficient subset. The great intuitive appeal of this definition led to its adoption by several authors, including Charnes, Cooper and Rhodes [1978] and Färe and Lovell [1978][6].

The Koopmans definition is more demanding than the Debreu-Farrell measure. Radial measures, such as DF_i, require that efficient observations belong to the isoquant, though not necessarily to the efficient subset. Consequently, the two concepts of technical efficiency are in conflict for any reference technology for which the isoquant diverges from the efficient subset. The ultimate importance of this issue is an empirical matter, but it is likely to be of greater concern when using nonparametric models (such as DEA). Several proposals have been made to ameliorate the problem in practice. We briefly examine two of them, but conclude that neither offers a very satisfying solution to the problem.

The first proposal (Bessent *et alii* [1988]) is to use constrained facet analysis (CFA) in DEA. If a conflict between the Koopmans definition and the radial efficiency measure is encountered, CFA maintains the radial approach to efficiency measurement, but modifies the reference technology. If an inefficient activity is not projected onto Eff $L(y)$ during DEA, CFA subsequently projects it onto the "nearest" hyperplane or facet belonging to Eff $L(y)$. For example, the radial input measure first projects observation 1 to the point 1' in Figure 2, then CFA extends the facet spanned by observations B and C and projects point 1' to point 1". The resulting efficiency measure no longer underestimates technical inefficiency.

Unfortunately, the solution offered by CFA is flawed. First, it confounds the specification of the set of production possibilities with the selection of an efficiency measure. The extension of a technology's hyperplanes is not justified by the assumptions of production analysis. In terms of Figure 2, one must be skeptical of the basis upon which point 1" is considered a "feasible" production possibility. Second, the

(6) Remark that Koopmans ([1951], pp. 60 and 80) actually requires the simultaneous membership of both the efficient subsets of the input and the output correspondences (see also Lovell [1993], p. 13), which amounts to membership in the graph efficient subset. But only for a few DEA reference technologies does membership in the input efficient subset imply that an observation is also in the output efficient subset (see, e.g., Färe, Grosskopf and Lovell ([1985], pp 46–47)). This is, however, not a limitation for our analysis as the input efficiency measures — to be discussed below — can be easily generalized to graph efficiency measures (along the lines of, e.g., Färe, Grosskopf and Lovell [1985]). An empirical application of radial and nonradial graph efficiency measures is provided in De Borger, Ferrier and Kerstens [1994].

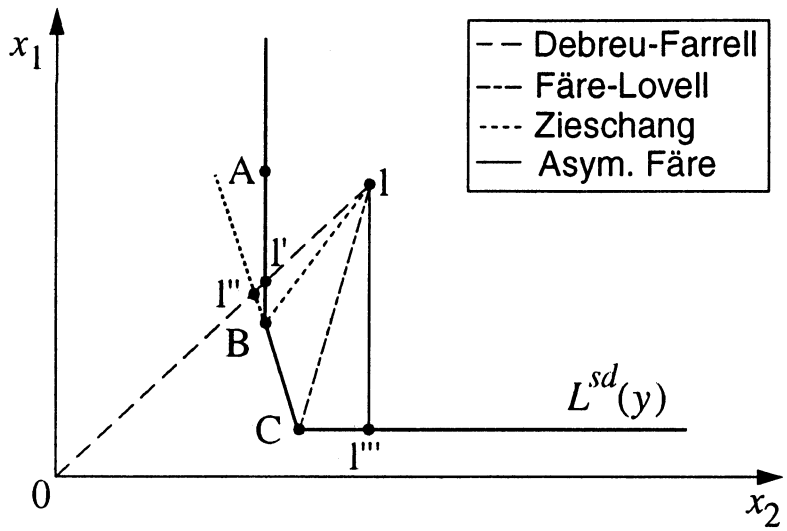

Figure 2
Constrained Facet Analysis and four measures of technical efficiency on a DEA input correspondence.

solution is not general. It requires that a "sufficient" number of efficient observations be available to define the "nearest" hyperplane belonging to Eff $L(y)$. This shortcoming of CFA was already recognized in the Bessent *et alii* [1988] article.

A second, indirect, solution to the divergence problem is to report any remaining slacks in addition to the radial efficiency measure itself (e.g., Lovell [1993] and Lovell and Vanden Eeckaut [1994]). Slacks, however, are rarely reported in the DEA literature. In a review of previous DEA studies, Bessent *et alii* [1988] found widespread occurrence of slacks. In one study, all of the inefficient units had additional slacks! While this suggestion provides all of the information needed to check whether an observation belongs to the efficient subset, it fails to summarize this information with a single number. Furthermore, it is worth mentioning the computational implications of obtaining correct information on slacks in the input and the output dimensions. Including a non-Archimedian constant in the objective function when computing the radial efficiency measure provides a good specification for the radial projection and yields correct information on slacks. But in practice this approach leads to numerical difficulties (see Ali and Seiford [1993], Charnes, Cooper and Rhodes [1978] and Färe and Hunsaker [1986]). Recently, Ali and Seiford [1993] find the maximal slacks for the radial input efficiency measure by solving a two stage linear programming DEA model. In the first step the traditional radial input efficiency measure is determined. In the second stage an additive model (as proposed in Charnes *et alii* [1985]) is calculated where the inputs of the observation being evaluated are now scaled down by the optimal radial efficiency measure obtained in the first step (see Appendix A for details).

In sum, the divergence between the intuitively appealing Koopmans definition of technical efficiency and the standard radial measurement of technical efficiency boils down to the issue of the appropriate economic interpretation of technical efficiency. Therefore, it is worthwhile reconsidering the use of radial efficiency measures in the mathematical programming approach. To that end, three of the alternative nonradial measures of technical efficiency are now considered.

3 Technical efficiency measures in the axiomatic literature

Färe and Lovell [1978] pioneered the axiomatic approach to technical efficiency measurement by proposing a set of desirable properties that an ideal measure of technical efficiency should satisfy. Given these properties, they proposed a new efficiency measure that compares inefficient observations to the efficient subset. We first review their contribution, then we present three nonradial measures of technical efficiency that have appeared in the literature, and finally we discuss the debate Färe and Lovell [1978] provoked. As above, the discussion concentrates on input efficiency.

Färe and Lovell [1978] suggested that a measure of technical efficiency should satisfy four properties. In the case of an input-based measure of technical efficiency, $E_i(x,y)$, these desirable properties can be formulated as follows:

1. Observed input vectors should be termed "efficient" if and only if they belong to the efficient subset:

$$\text{If } x \in L(y),\ y > 0,\ \text{then } E_i(x,y) = 1 \Leftrightarrow x \in \text{Eff}\, L(y);$$

2. Inefficient input vectors should be compared to vectors belonging to the efficient subset:

$$\text{If } x \in L(y),\ y > 0, \text{ and } x \notin \text{Eff}\, L(y),$$
$$\text{then } E_i(x,y) \text{ should compare } x \text{ to some } x^* \in \text{Eff}\, L(y);$$

3. $E_i(x,y)$ should be homogeneous of degree minus one (e.g., a doubling of the input vector used by the inefficient observation halves

the efficiency measure):

$$\begin{aligned} &\text{If } x \in L(y), \text{ and } \lambda x \in L(y),\ y > 0, \\ &\text{then } E_i(\lambda x, y) = \lambda^{-1} E_i(x, y), \\ &\text{for all } \lambda \in [\lambda^0, +\infty), \text{ where } \lambda^0 x \in \operatorname{Isoq} L(y); \end{aligned}$$

4. $E_i(x,y)$ should be strictly negatively monotonic (i.e., increasing one input, while holding all other inputs and all outputs constant, lowers the efficiency measure):

$$\text{If } x \in L(y),\ y > 0, \text{ and } x' \geqslant x, \text{ then } E_i(x,y) > E_i(x',y).$$

The first two properties say that $E_i(x,y)$ should conform to the Koopmans definition of efficiency. They express the desire to scale inefficient vectors to an element of the efficient subset. Sensitivity of the efficiency measure with respect to input usage is addressed by the third and fourth properties. The third one imposes a direct proportionality between inputs used in all dimensions and technical efficiency, while the fourth guarantees sensitivity of input usage in any single dimension.

The Färe and Lovell [1978] article led to proposals of further desirable properties and additional nonradial efficiency measures. This literature, and our discussion, focusses on the properties of four particular measures of technical efficiency: the Debreu-Farrell measure (DF_i), the Färe-Lovell measure (FL_i), the Zieschang [1984] measure (Z_i), and the asymmetric Färe measure (AF_i) (Färe [1975], and Färe, Lovell and Zieschang [1983])[7]. The Debreu-Farrell input measure of technical efficiency was defined in the previous section; the others are now presented.

Radial efficiency measures do not necessarily compare the observations to the efficient subset and therefore fail to satisfy the first two axioms[8]. Färe and Lovell [1978] propose a measure of technical efficiency, known as the Färe-Lovell, or Russell, measure, that does not

(7) Färe [1975] first proposed the asymmetric Färe measure, calling it an input efficiency function. Färe, Lovell and Zieschang [1983] refer to it as the overall asymmetric measure of technical efficiency. Analogous to the other efficiency measures, we refer to the measure by its originator.

(8) The weak measure of technical efficiency (see Färe, Grosskopf and Lovell [1985]) is not discussed in this paper as it only differs from the Debreu-Farrell efficiency measure in that the weak efficient subset is used as a reference set instead of the isoquant.

possess this shortcoming[9]. The Färe-Lovell input technical efficiency measure is given by:

$$FL_i(x,y) = \min\left\{\sum_{i=1}^{m}\frac{\lambda_i}{m} \;\middle|\; (\lambda_1 x_1,\ldots,\lambda_m x_m) \in L(y),\ \lambda_i \in (0,1]\ \forall i\right\}$$

FL_i minimizes the arithmetic mean of the proportional reductions in each input dimension (i.e., the scalars λ_i). A nonradial measure, FL_i scales each input by a different proportion. To determine the projection point each of its inputs must be contracted with the corresponding scalar element of the efficiency measure $(\lambda_1 x_1,\ldots,\lambda_m x_m)$.

The Zieschang measure of input technical efficiency is:

$$Z_i(x,y) \;=\; FL_i\left(x\cdot DF_i^+(x,y),y\right)\cdot DF_i^+(x,y),$$

where

$$DF_i^+(x,y) \;=\; \min\left\{\lambda \mid \lambda \geqslant 0,\ \lambda x \in L^+(y) = L(y) + \mathbb{R}_+^m\right\}.$$

Z_i is an amalgamation of the Debreu-Farrell (this component is denoted λ^{z-df}) and Färe-Lovell measures (with typical element λ_i^{z-fl}). It radially scales the inefficient observation down to the isoquant, and then shrinks the resulting input vector until it reaches an element of the efficient subset. Note that the nonradial component of Z_i is calculated on a technology satisfying strong input disposability, while the (radial) Debreu-Farrell measure is defined without this restriction on the technology. The projection point associated with the Zieschang measure is found by rescaling each input dimension by the two components of Z_i; i.e., Z_i is actually a multiplicative measure, with the projection point given by $(\lambda^{z-df}\lambda_1^{z-fl}x_1,\ldots,\lambda^{z-df}\lambda_m^{z-fl}x_m)$.

Finally, the asymmetric Färe technical efficiency measure is:

$$AF_i(x,y) \;=\; \min\left\{AF_i^j(x,y)\right\} \quad , \quad j = 1,\ldots,m,$$

where

$$\begin{aligned}
AF_i^1(x,y) &= \min\left\{\lambda_1 \mid (\lambda_1 x_1,\ldots,x_j,\ldots,x_m) \in L(y),\ \lambda_1 \in (0,1]\right\}\\
&\ \vdots\\
AF_i^j(x,y) &= \min\left\{\lambda_j \mid (x_1,\ldots,\lambda_j x_j,\ldots,x_m) \in L(y),\ \lambda_j \in (0,1]\right\}\\
&\ \vdots\\
AF_i^m(x,y) &= \min\left\{\lambda_m \mid (x_1,\ldots,x_j,\ldots,\lambda_m x_m) \in L(y),\ \lambda_m \in (0,1]\right\}.
\end{aligned}$$

(9) The term "Russell measure" is a misnomer. First, because the measure appears originally in Färe and Lovell [1978]. Second and foremost, because Russell himself clearly advocates the Debreu-Farrell efficiency measure (see Russell [1985, 1988]). We therefore prefer the moniker "Färe-Lovell measure".

AF_i scales down each input in turn, while holding all other inputs fixed. It then takes the minimum over these scalings as its measure of technical efficiency. This measure scales an inefficient observation down to the boundary of $L(y)$, which does not necessarily coincide with either the isoquant, the weak efficient set, or the efficient subset. Its projection point is found by scaling down the single input dimension for which the corresponding scalar element minimizes the vector of components. If the j-th component is the minimum (λ_j), then the projection point is $(x_1, \ldots, \lambda_j x_j, \ldots, x_m)$.

A few remarks can be made on the connections between these efficiency measures. A first remark is on the relations among these efficiency measures. The Färe-Lovell measure generalizes both the Debreu-Farrell and the asymmetric Färe measures. For $\lambda_1 = \lambda_2 = \cdots = \lambda_m$, FL collapses to DF_i; and for $\lambda_i = 1$ for $AF_i^j \neq \min AF_i$, FL_i reduces to AF_i. Furthermore, in the case of a single input dimension $(m = 1)$ these four efficiency measures all coincide $(DF_i = FL_i = Z_i = AF_i)$. More importantly, for a given reference technology these four efficiency measures can be a priori ordered as follows:

$$DF_i \geqslant Z_i \geqslant FL_i \geqslant AF_i .$$

This complete ranking received so far little notice in the literature[10].

A second remark is on the similarity between DF_i and Z_i. The Zieschang efficiency measure is defined with the intent of eliminating slacks. Thus, DF_i coincides with Z_i if it scales an inefficient observation down to the efficient subset. If the radial measure projects an inefficient observation to another subset, however, slack will remain and it will not coincide with the Zieschang measure. As a final remark, strictly speaking, the above definitions hold only if none of the inputs is zero. If the input vector is semi-positive, then the definitions of the Färe-Lovell measure (and consequently the Zieschang) and asymmetric Färe measures need to be adjusted. The idea behind the modifications proposed in Färe, Lovell and Zieschang [1983] is to avoid any influence of zero dimensions on the efficiency measure. As a consequence, the performance of different observations may possibly be gauged relative to spaces of different dimensionality. This seems clearly unsatisfactory. However, since this issue does not affect our empirical illustration, we simply conclude that the treatment of zeros warrants more attention in future work.

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(10) Färe, Lovell and Zieschang [1983] demonstrate many of these relations. The relative magnitudes of FL_i and Z_i, however, were only recently proven in Kerstens and Vanden Eeckaut [1994].

Figure 2, presented above, illustrates these four efficiency measures on the DEA input correspondence. For clarity, the figure is designed such that each efficiency measure selects a different reference point, though this need not be the case. The radial efficiency measure, DF_i, calculates technical inefficiency along a ray through the origin, which leaves slack in the first input (i.e., the distance from 1' to B in Figure 2). The Färe-Lovell measure, FL_i, scales the inefficient observation down to observation C. The Zieschang measure, Z_i, adjusting the radial efficiency measure for the remaining slack in the first input, relates the inefficient observation to point B. Finally, because performance is worst in the first input dimension, the asymmetric Färe efficiency measure selects point $1'''$ as the reference point. This leaves slack in the second input (i.e., the distance from $1'''$ to C).

These four efficiency measures are thoroughly discussed in the theoretical debate that followed Färe and Lovell [1978]. This literature concludes that for a broad class of reference technologies none of these measures can satisfy all four ideal conditions simultaneously (see Färe, Lovell and Zieschang [1983], Russell [1988], and Zieschang [1984]). More generally, Bol [1986] and Russell [1988] show that no measure of technical efficiency can satisfy all four conditions simultaneously for the broad class of technologies considered in this literature. This finding changes the focus of the debate to the consideration of properties that are both desirable and feasible. Bol [1986] suggests two possibilities. First, narrow the class of reference technologies to which the efficiency measures are applied(11). Second, relax (or drop) at least one of the four initial ideal conditions and/or add other desirable properties to the list. But this debate so far yielded few clear-cut results.

From a practitioner's viewpoint it is important to stress that the negative results obtained in this theoretical literature require some qualification if one no longer focuses on a broad class of reference technologies. Although the problem of the choice among efficiency measures does not disappear on a restricted production technology, the list of satisfied properties changes if efficiency measurement is restricted to popular nonparametric reference technologies such as DEA(12). In DEA models DF_i is homogeneous of degree minus one, but fails to satisfy the first two ideal conditions and only satisfies a weaker version

(11) This strategy was partly pursued by Färe, Lovell and Zieschang [1983].

(12) See Kerstens and Vanden Eeckaut [1994], who also consider the properties satisfied on the FDH.

of the fourth condition (i e., it is weakly monotonic[13]). FL_i satisfies the first two and the fourth conditions, but only a weaker version of the third axiom. It is subhomogeneous, rather than homogeneous, of degree minus one (see Petersen [1990])[14]. Z_i satisfies all four axioms on DEA. Finally, AF_i satisfies the first condition, but fails to meet the other conditions. While it correctly identifies input vectors as efficient if they belong to the efficient subset, in general, the inefficient input vectors are not compared to this subset. AF_i also satisfies only subhomogeneity of degree minus one and weak negative monotonicity. Consequently, only Z_i satisfies all four ideal conditions of an efficiency measure on DEA. While FL_i manages to meet three out of the four standards, both DF_i and AF_i only fulfill one of the four axioms on DEA.

The theoretical debate as to the "best" measure of technical efficiency is clearly inconclusive. None of the proposed measures enjoys theoretical superiority over the others. This in part may explain why practitioners have ignored the debate and continue to use the radial measures as their standard for efficiency evaluation. In addition, there are two arguments that appear in the margin of the axiomatic literature that favor the radial measures (see Lovell and Schmidt [1988]). The first argument in support of the radial measures is that they have a straightforward cost interpretation[15]. In fact, the Debreu-Farrell measure can be written as the ratio of minimal to actual costs:

$$DF_i(x,y) = \lambda = \frac{w^t(\lambda x)}{w^t x},$$

where w is a vector of input prices. Thus, it shows the potential cost reduction resulting from the elimination of technical inefficiency. Note that scaling all factor prices equally, or any one factor price individually, does not affect the cost ratio. Factor price independence is due to the radial nature of the measurement. No such straightforward cost interpretation is available for the nonradial efficiency measures

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(13) Weak monotonicity is defined as:

$$\text{If } x \in L(y),\ y > 0, \text{ and } x' \geqslant x, \text{ then } E_i(x,y) \geqslant E_i(x',y).$$

That is, increasing at least one of the inputs, while holding all others constant, cannot increase the efficiency measure.

(14) Subhomogeneity of degree minus one is defined by:

$$\text{If } x \in L(y), \text{ and } \lambda x \in L(y),\ y > 0, \text{ then } E_i(\lambda x, y) \lesseqgtr \lambda^{-1} E_i(x,y) \text{ if } \lambda \gtreqless 1.$$

That is, the scaling of the input vector by a factor larger (smaller) than unity leads to an efficiency measure smaller (larger) than the inverse scaling of the efficiency measure by the same factor.

(15) This point is noted by Debreu [1951], and stressed by Russell [1985].

— cost ratios are factor price dependent, and therefore, physical and cost efficiency need not coincide in this case[16]. This is an important limitation of the nonradial efficiency measures. It must be added, however, that the factor price independent cost interpretation of the radial efficiency measures is not always valid. For example, if additional behavioral constraints are imposed on the production technology (e.g., due to rate-of-return regulation, as in Färe and Logan [1992]), its cost interpretation also becomes factor price dependent.

A second, more theoretical, argument in favor of the radial efficiency measure is that there is an equivalence between it and the input correspondence set $L(y)$ (see Lovell and Schmidt [1988] and Lovell [1993]). As the inverse of the input distance function, the input-based radial efficiency measure contains all of the information on the underlying technology $L(y)$:

$$L(y) = \{x \mid 0 < DF_i(x,y) \leqslant 1\}.$$

Furthermore, DF_i gives a functional representation of the isoquant:

$$\operatorname{Isoq} L(y) = \{x \mid DF_i(x,y) = 1\}.$$

However, a similar argument can be made in favor of all the nonradial efficiency measures. This reasoning is developed explicitly for the case of the asymmetric Färe efficiency measure. The asymmetric Färe efficiency measure contains all information on the technology represented by any input correspondence satisfying strong input disposability (Färe ([1975], p. 322) and Färe, Lovell and Zieschang ([1983], p. 167)):

$$L^+(y) = \{x \mid 0 < AF_i(x,y) \leqslant 1\}.$$

Moreover, AF_i provides a functional representation of the efficient subset of a general input correspondence $L(y)$:[17]

$$\operatorname{Eff} L(y) = \{x \mid AF_i(x,y) = 1\}.$$

For the other nonradial alternatives, i.e. FL_i and Z_i, this analogy between on the one hand the efficiency measure and the input correspondence with strong input disposability $(L^+(y))$, and on the other hand the efficiency measure and the efficient subset of $L(y)$ holds true as well. In general, this equivalence is valid for any efficiency measure

(16) See Kopp [1981a, b] for discussions of the cost interpretations of the Färe-Lovell and the asymmetric Färe efficiency measures.

(17) This result holds true for a general input correspondence because $L(y)$ and $L^+(y)$ have the same efficient subset (Färe [1975] p. 321).

satisfying the first axiom. Therefore, if one adopts the Koopmans definition of technical efficiency, then the efficient subset, not the isoquant, is relevant and nonradial efficiency measures are favored over radial efficiency measures.

It is unclear whether these additional arguments bestow preferred status to any of the efficiency measures discussed above. What emerges from the theoretical debate on efficiency measures is that both radial and nonradial measures possess certain advantages and disadvantages relative to one another. We therefore turn our attention to an empirical analysis in order to ascertain the relative performances of these measures in practice.

4 An empirical comparison of efficiency measures for US banks using DEA

4.1 Description of the sample

This section systematically explores whether the choice among the various efficiency measures makes any difference in practice. The technical efficiency of a sample of 575 US banks operating in 1984 is calculated using the four efficiency measures described above on a DEA reference technology. The results across efficiency measures are then compared. To the best of our knowledge, an empirical comparison of radial and nonradial efficiency measures does not appear in the DEA literature.

A first step in assessing productive efficiency is to define the inputs and outputs involved in the transformation process. This is an uncertain first step when measuring banking efficiency, because considerable disagreement exists as to the appropriate definition of bank inputs and outputs. Prior research on efficiency in banking has distinguished two approaches to input/output definition and measurement — the "intermediation" and the "production" approaches. Each of these approaches has its advantages and drawbacks and no consensus exists as to the most appropriate way to analyze the complexity of banking activities[18].

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[18] The intermediation approach views banks as intermediaries between lenders and borrowers — banks accumulate deposits and purchased funds and transform these funds into financial services. Under this approach, outputs are measured in monetary volumes because the inputs not only include traditional inputs (i.e., labor, capital and materials), but also the interest costs of purchased funds. Under the production approach, banks are viewed as producers of deposit and loan services using traditional inputs. In this setting, outputs are measured by the number of different deposit and loan accounts serviced, or by the numbers of transactions performed on each

Table 1: Descriptive statistics on the sample of US banks (N=575).

Inputs / Outputs	Mean	Standard Deviation	Minimum Value	Maximum Value
x_1	111.17	130.0	5.10	1165.79
x_2	533145.05	790336.7	2260.13	7608838
x_3	1034901.77	1372993.0	36806.48	1155379.05
y_1	12334.50	15819.4	136	151029
y_2	25470.81	34238.0	226	404045
y_3	2764.97	23965.5	0	570385
y_4	5949.33	10332.9	0	151828
y_5	1476.99	3822.2	0	84515

Both approaches have been used in the recent empirical literature on bank efficiency: Aly, Grabowski, Pasurka and Rangan [1990], Berger, Hanweck and Humphrey [1987] and Berger and Humphrey [1991] employ the intermediation approach; while Ferrier and Lovell [1990], Fried, Lovell and Vanden Eeckaut [1993] and Tulkens [1993] opt for the production approach. This literature also uses a variety of reference technologies. For example, Aly, Grabowski, Pasurka and Rangan [1990] use variable returns to scale DEA; Ferrier and Lovell [1990] use both stochastic parametric frontiers and DEA; Fried, Lovell and Vanden Eeckaut [1993] choose the FDH approach; Tulkens [1993] compares DEA and FDH. Surveying the empirical literature, a common finding is that technical efficiency has a greater effect on banks' observed costs than do other types of inefficiencies or economies of scale or scope (see Berger, Hunter and Timme [1993] and Colwell and Davis [1992] for recent reviews of this literature).

Our analysis adopts the production approach. Five outputs are specified: the numbers of demand (y_1) and time (y_2) deposit accounts, and the numbers of real estate (y_3), instalment (y_4) and commercial (y_5) loans. The outputs are measured in terms of numbers of accounts. Three inputs are considered: the total number of employees (x_1), occupancy and equipment costs (x_2), and expenditures on materials (x_3)[(19)]. Descriptive statistics of these variables are given in Table 1. The data were collected under the Federal Reserve System's Functional Cost Analysis

output. See the discussion in Colwell and Davis [1992].

[(19)] Note that the definition of the quantities of capital (x_2) and materials (x_3) is not ideal, but no better information on these inputs is available.

(FCA) program, a program designed to assist participating banks to increase their operating efficiency by providing them with average performance figures for banks with similar characteristics against which to compare their own performance. This feature of the FCA program helps to ensure that banks have a self-interest in reporting data accurately.

A variable returns to scale DEA model with strong input and output disposability serves as the reference technology in measuring the input-based technical efficiency of banks using the four measure discussed above. This reference technology is given by the following input correspondence:

$$L(y)^{sd-vrs} = \left\{x \mid \mathbf{Y}^t z \geqq y,\ \mathbf{X}^t z \leqq x,\ \mathbf{I}_k^t z = 1,\ z \in \mathbb{R}_+^k\right\},$$

where $\mathbf{Y}$ is the $k \times n$ matrix of observed outputs, $\mathbf{X}$ is the $k \times m$ matrix of observed inputs, z is a $k \times 1$ vector of intensity, or activity, variables, and y and x are $n \times 1$ and $m \times 1$ vectors of outputs and inputs, respectively, and $\mathbf{I}_k$ is a $k \times 1$ unity vector. Remark that a weakly disposable DEA production technology with variable returns to scale has the same efficient subset. Since the paper focuses on the distinction between the efficient subset and the other subsets of a production technology, the empirical application is based on a traditional DEA model assuming strong input and output disposability.

As to the computational aspects, we briefly indicate how the nonradial efficiency measures can be obtained for the above DEA model. First, the Färe-Lovell efficiency measure results from solving a single linear program for each observation, which allows each input dimension in the constaints to be proportionally reduced by a scalar λ and which minimizes the arithmetic mean of these scalars in the objective function. Second, the Zieschang efficiency measure involves the solution of two linear programs for each observation. In the first step one calculates the traditional radial input efficiency measure DF_i, and in the second step the Färe-Lovell efficiency measure FL_i is computed for the same observation rescaled by DF_i. Finally, the asymmetric Färe efficiency measure requires solving m linear programs —one for each input dimension— for each observation, and taking the minimum over the m resulting optimal values. All details on the linear programs needed to implement the radial and especially the nonradial efficiency measures are discussed in Appendix A.

4.2 Empirical results

We assess the four input efficiency measures as defined above on this reference technology in the following way. First, the cardinality of the various subsets is discussed. Then the problem of slacks in the radial input efficiency measure is illustrated. Further, descriptive statistics and correlations between the various efficiency measures are described.

Of the 575 US banks in the sample, 546 are judged to be inefficient. All 29 efficient observations belong to the efficient subset. Only 134 out of the 546 inefficient observations, however, are projected onto the efficient subset by the radial efficiency measure. That is, only about a quarter of the inefficient observations have zero slacks in their inputs. This result confirms the potential importance of slacks for radial measurement on the widely used DEA models.

The importance of slacks is illustrated for the radial input measure in Table 2. To facilitate comparison, the slack in each dimension is expressed as a percentage of the observed inputs and outputs for each bank being evaluated.

Table 2: Slacks and radial efficiency in the inputs on DEA (N=546).

Dimension	Mean	Standard Deviation	Minimum Value	Maximum Value
Total Slack (%)				
Input 1	72.11	16.69	0.14	96.66
Input 2	67.35	16.43	0.31	93.28
Input 3	68.58	17.72	0.14	98.83
Output 1	2.47	26.52	0	555.3
Output 2	8.61	30.13	0	373.2
Output 3	13.50	68.02	0	817.5
Output 4	83.03	309.53	0	6204.0
Output 5	110.89	1571.20	0	36090.0
Slack eliminated by the radial input efficiency measure (%)				
All inputs	66.38	17.66	0.14	93.28
Slack not eliminated by the radial input efficiency measure (%)				
Input 1	5.73	9.05	0	68.13
Input 2	0.97	4.65	0	45.80
Input 3	2.20	5.10	0	59.35

Total slack per dimension is the difference between the observed data point and the projection point; i.e., the linear combination determined by the non-zero components in z^*. Table 2 reveals the importance of this phenomenon. Total slack is partially eliminated by the radial input efficiency measure (termed radial slack). The remaining slack (termed nonradial slack) is, on the average, of minor importance for the second input dimension. Considering the wide range of values, however, nonradial slack is far from negligible, especially in the first and third input dimensions[20].

An examination of the empirical distributions of the four efficiency measures reveals their similarities and differences. Table 3 contains the descriptive statistics of the distributions for both the full sample and for the inefficient observations only. The following observations can

Table 3: Input efficiency measures on DEA.

$E_i(x,y)$	Mean	Standard Deviation	Skewness	Kurtosis	Minimum Value	Maximum Value
All Observations (N = 575)						
DF_i	.370	.225	1.513	4.680	.067	1.000
FL_i	.328	.215	1.809	5.940	.036	1.000
Z_i	.341	.218	1.657	5.343	.037	1.000
AF_i	.134	.227	3.021	11.406	.006	1.000
Inefficient Observations Only (N = 546)						
DF_i	.336	.177	1.465	5.315	.067	.999
FL_i	.292	.152	1.472	5.524	.036	.951
Z_i	.306	.161	1.374	5.053	.037	.955
AF_i	.088	.110	3.223	16.694	.006	.855

be made. First, the mean efficiency levels are quite low and are much lower than the radial results reported in Ferrier and Lovell [1990]. They are also below the averages of about .80 which are typically reported in most bank efficiency studies based on DEA models (see Berger, Hunter and Timme [1993] and Colwell and Davis [1992]). Second, the radial efficiency measure had the largest mean, followed by the Zieschang, the Färe-Lovell, and the asymmetric Färe efficiency measures, reflecting the complete ordering among efficiency measures pointed out earlier. Third, the efficiency distributions have large ranges, are positively skewed and have positive kurtoses. This results in long, fat right-tailed distributions as compared with the normal distribution. Fourth, the standard deviations are very similar over the various efficiency measures. Finally, the extreme nature of the asymmetric Färe efficiency measure is evident.

[20] The results for the same sample on FDH differ in that the radial efficiency measure eliminates, relatively speaking, less slacks: see De Borger, Ferrier and Kerstens [1994].

Figure 3 illustrates the density distributions of the four efficiency measures. The nature of the resulting efficiency distributions is formally evaluated using two nonparametric test statistics (see Siegel and Castellan [1988]). First, a Friedman two-way analysis of variance based on ranks was computed to test whether the efficiency measures are all drawn from the same population (or at least from populations with the same median). Second, a Wilcoxon signed rank test does the same for all pairs of efficiency measures. These tests suggest that the efficiency measures are not drawn from a single population ($Fr. = 1437.84 > \chi^2_{0.001}(3) = 16.27$), and that no pair of efficiency measures shares a common distribution[21].

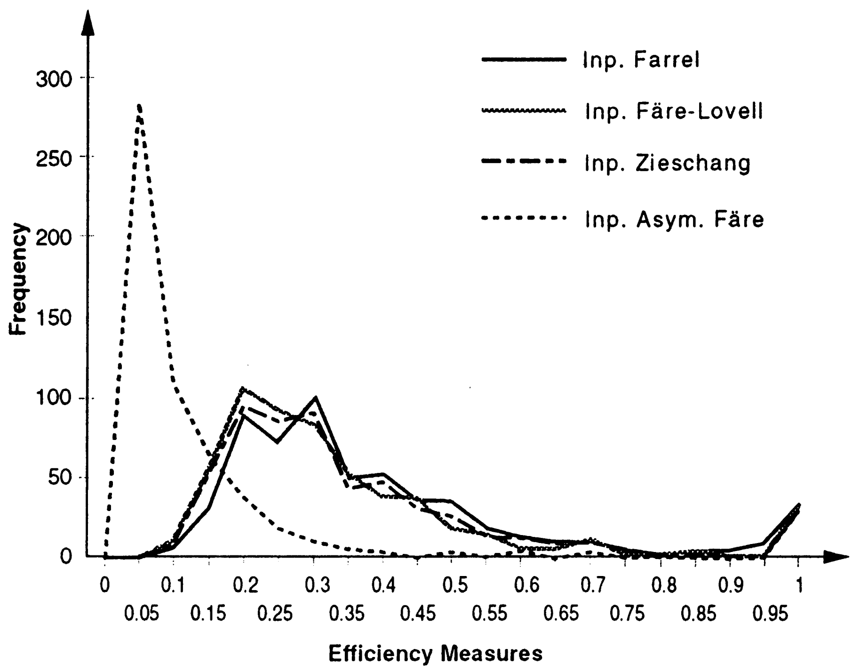

Figure 3: Densities of input efficiency measures on a DEA model (N = 575).

The low average efficiency scores require some comments. First and foremost, this difference is accounted for by the fact that Ferrier and Lovell [1990] also included a large set of environmental variables in their analysis. Under the nonparametric approach to efficiency measurement, increasing the number of dimensions reduces the amount of inefficiency detected. The impact of the dimensionality of the production technology on radial and nonradial efficiency measures is discussed

(21) For the sake of brevity, details on the latter test statistics are suppressed.

in Kerstens and Vanden Eeckaut [1994]. To accentuate the differences across the four efficiency measures considered in this paper, the dimensionality of the production technology is kept to a minimum. Therefore, only input and output variables are included in the model, and environmental variables are ignored. Second, another plausible cause for the results is the presence of a large variation in the relative specialization of the banks in the sample. There are two arguments. First, the sample contains 52 inefficient observations with zero values for one or more of the outputs. Furthermore, 8 out of 29 efficient observations have at least a single zero output (details are in Appendix B). Second, in another study by Ferrier *et alii* [1993] the same sample has been examined for the existence of economies of diversification. It turned out that on average there are cost advantages for specialized banks, as the majority of observations exhibited diseconomies of diversification[22].

In addition to the distribution of these efficiency measures, it is worthwhile to look at the individual dimensions underlying each of them, especially for the inefficient observations. Scaling the input vectors of an inefficient observation with each of these components yields its projection point or role model. Descriptive statistics for each dimension of the nonradial efficiency measures are reported in Table 4 for the inefficient observations only. To be specific, for FL_i these dimensions are $(\lambda_1, \ldots, \lambda_m)$, for Z_i these are $(\lambda^{z-df}\lambda_1^{z-fl}, \ldots, \lambda^{z-df}\lambda_m^{z-fl})$, and for AF_i the components are $(1, \ldots, \lambda_j, \ldots, 1)$ if the minimum of this efficiency measure is in the j-th dimension. The number of observations on which the statistics are based (see the last column) is identical, except for the asymmetric Färe efficiency measure. Recall that for the latter only a single dimension of the inefficient observation is scaled down. Results are easier to interpret if the computations for each dimension are based on the inefficient observations which are scaled down in this particular dimension, instead of including the other observations which have unity components for that dimension. Note, that the radial efficiency measure is, of course, identical for each dimension and that is it only included to facilitate comparisons.

It turns out that the components of the nonradial efficiency measures agree both on the dimensions with the least and worst technical inefficiency: on average the least technical inefficiency is detected in the first input dimension, while performance is worst in the second input

(22) On FDH the average of the distributions of these efficiency measures is much higher. It is in no way unusual relative to the results reported in other applications: see De Borger, Ferrier and Kerstens [1994]. This divergence of results on DEA and FDH for the same sample illustrates the major impact of the convexity assumption, especially in the presence of many specialized observations.

Table 4
Components of the input efficiency measures on DEA (inefficient observations only).

$E_i(x,y)$	Mean	Standard Deviation	Skewness	Kurtosis	Minimum Value	Maximum Value	N
DF_i	.370	.225	1.513	4.680	.067	1.000	546
FL_i Input 1	.301	.171	1.852	6.808	.067	1.000	546
FL_i Input 2	.277	.173	1.719	6.516	.032	1.000	546
FL_i Input 3	.297	.184	1.590	5.913	.010	1.000	546
Z_i Input 1	.326	.164	1.427	5.426	.067	.997	546
Z_i Input 2	.278	.166	1.649	6.250	.033	.999	546
Z_i Input 3	.314	.177	1.353	5.118	.012	.999	546
AF_i Input 1	.325	.191	1.450	4.105	.158	.855	21
AF_i Input 2	.071	.084	3.761	24.631	.006	.827	489
AF_i Input 3	.177	.151	1.485	4.342	.022	.598	36

dimension. The latter result is also evident from the largest number of observations being scaled down in that dimension by the asymmetric Färe efficiency score. This variation in the distributions of the components of each nonradial efficiency score is also confirmed by a series of nonparametric test statistics. First, a Friedman two-way analysis of variance based on ranks reveals that the components of none of the nonradial efficiency measures follow the same distribution. Furthermore, for no single dimension do these efficiency measures share a common distribution. Second, a Wilcoxon signed rank test confirms these negative results, except for some similarities between components of FL_i and Z_i and between the first and third dimension of AF_i. Overall, these results seem to indicate that there are considerable differences in the proposed role models between, on the one hand, the radial and nonradial efficiency measures, and on the other hand among the nonradial efficiency measures(23).

Similarities in efficiency rankings across the four measures can be inferred from the matrix of Pearson product-moment correlation coefficients reported in Table 5. Note that all correlation coefficients

(23) Because of space limitations, details on both nonparametric test statistics are suppressed.

Table 5: Correlation matrix between input efficiency measures on DEA.

$E_i(x,y)$	DF_i	FL_i	Z_i	AF_i
All Observations (N = 575)				
DF_i	1.000	0.979	0.985	0.872
FL_i		1.000	0.994	0.919
Z_i			1.000	0.905
AF_i				1.000
Inefficient Observations Only (N = 546)				
DF_i	1.000	0.972	0.976	0.839
FL_i		1.000	0.990	0.864
Z_i			1.000	0.858
AF_i				1.000

are significantly different from zero at the 99% significance level. Given the large number of efficient observations, the positive correlations are very high. Therefore, we added the correlation matrix calculated on the inefficient observations only. The size of the correlations is aided by the fact that for the 134 inefficient observations that are projected onto the efficient subset, the radial and Zieschang efficiency measures are equal. The Färe-Lovell and Zieschang measures also coincide for 71 (about 13%) inefficient observations. The other efficiency measures do not agree for any of the observations. The highest correlations found are those between the Färe-Lovell and Zieschang efficiency measures. This finding is not too surprising, given that the latter incorporates the former in its definition (and, as indicated above, sometimes selects the same projection point). The lowest correlations in the matrix are those between the radial and the asymmetric Färe efficiency measures. Furthermore, the asymmetric Färe measure correlates only slightly better with the Färe-Lovell and the Zieschang efficiency measure[(24)].

5 Summary and conclusions

The purpose of this paper was twofold. First, the choice of a technical efficiency measure was analyzed from a theoretical perspective. A

[(24)] These results are similar to those obtained on the FDH, except that on the latter reference technology all correlation coefficients are lower: see De Borger, Ferrier and Kerstens [1994].

review of the axiomatic literature provided a list of desirable properties and three alternatives to the radial efficiency measure. Unfortunately, neither the radial nor the nonradial efficiency measures meet all of the desirable properties for a large class of production technologies. The primary disadvantage of the radial measure is its failure to conform to the Koopmans definition of technical efficiency. Each of the nonradial measures conforms to the Koopmans definition of efficiency, and are thus superior to the radial measure on this score. However, none of the nonradial measures satisfies all of the properties of an "ideal" efficiency measures, therefore it is difficult to choose among them on theoretical grounds alone. In addition, the radial measure possesses an attractive factor price independent cost interpretation vis-à-vis the nonradial measuring, further complicating the choice among alternatives. For a specific reference technology like DEA, however, the picture changes somewhat, as the Zieschang efficiency measure satisfies all four Färe and Lovell [1978] axioms while the Färe-Lovell efficiency index meets three of these properties. In this case the radial and the asymmetric Färe efficiency measures satisfy only a single axiom.

Second, to provide a comparison of their empirical performances, these four technical efficiency measures were applied to a set of input and output data on US banks using a DEA model. Wide differences in the distributions of technical efficiency and in the resulting correlations were revealed by this exercise. It also showed that the radial efficiency measure is not a good empirical substitute for the nonradial alternatives, as on average it scaled inefficient observations down to projection points far removed from the efficient subset.

Two final conclusions emerge from this theoretical and empirical analysis. First, the choice among the various alternative measures of technical efficiency is an important consideration within the DEA framework. Empirical evidence confirms the theoretical intuition that the radial efficiency measure does a poor job in closing the distance between the inefficient observations and the efficient subset. Second, both the theoretical arguments and empirical evidence suggest that the Färe-Lovell and Zieschang efficiency measures provide valuable alternatives to radial measurement. These measures appear to differ little in either theory or in practice.

APPENDIX A
Computation of efficiency measures on DEA

This appendix first discusses the linear programs developed to compute the radial efficiency measure in the inputs and the optimal slack vectors. Then it proceeds to the calculation of the nonradial alternatives.

To calculate the radial efficiency measure in the inputs $(DF_i(x,y))$ on this deterministic non-parametric reference technology requires solving the following linear program for each observation (x^0, y^0) being evaluated:

$$\begin{aligned} &\min_{\lambda, z_i} \lambda \\ \text{s.t.} \quad & \sum_{i=1}^{k} y_{ij} z_i \geqq y_{0j} \qquad \text{for } j = 1, \ldots, n \\ & \sum_{i=1}^{k} x_{i\ell} z_i \leqq \lambda x_{0\ell} \qquad \text{for } \ell = 1, \ldots, m \\ & \sum_{i=1}^{k} z_i = 1 \\ & \lambda \geqq 0, \quad z_i \geqq 0 \qquad \text{for } i = 1, \ldots, k. \end{aligned}$$

Or, in a more compact matrix notation and introducing explicitly slack vectors to convert the above inequality constraints into equalities leads to the following formulation:

$$\begin{aligned} &\min_{\lambda, z, s, e} \ \lambda \\ \text{s.t.} \quad & \mathbf{Y}^t z - s = y^0 \\ & x^0 \lambda - \mathbf{X}^t z - e = 0 \\ & \mathbf{I}_k^t z = 1 \\ & \lambda \geqq 0, \ z \geqq 0, \ s \geqq 0, \ e \geqq 0, \end{aligned}$$

where s and e are vectors of surplus outputs and excess inputs of dimension n and m respectively. In the second stage the maximal slacks and the optimal activity vector are obtained by solving one additional linear program for each observation (x^0, y^0):

$$\begin{aligned} &\min_{z, s, e} -\mathbf{I}_n^t s - \mathbf{I}_m^t e \\ \text{s.t.} \quad & \mathbf{Y}^t z - s = y^0 \\ & -\mathbf{X}^t z - e = -\lambda^* x^0 \\ & \mathbf{I}_k^t z = 1 \\ & z \geqq 0, \ s \geqq 0, \ e \geqq 0, \end{aligned}$$

where λ^* is the optimal radial efficiency measure determined in the first stage, i.e. the previous linear program. If one is only interested in the optimal values of the efficiency measure, this second step is redundant.

To compute the nonradial alternative efficiency measures requires some simple modifications of the above presented linear programs. Their only drawback is their sometimes higher computational cost, as they can involve the solution of more than one linear program for each observation. The discussion starts with the Färe-Lovell efficiency measure, then deals with the Zieschang efficiency measure, and finally mentions the computation of the asymmetric Färe efficiency measure.

The calculate the Färe-Lovell efficiency measure in the inputs $FL_i(x, y)$ requires solving the following linear program for each observation (x^0, y^0):

$$\begin{aligned} \operatorname*{Min}_{\lambda,z} \frac{1}{m}\mathbf{I}_m^t\lambda & \\ s.t. \qquad \mathbf{Y}^t z &\geqq y^0 \\ x^0 \odot \lambda - \mathbf{X}^t z &\geqq 0 \\ \mathbf{I}_k^t z &= 1 \\ \lambda &\leqq 1 \\ \lambda \geqq 0,\ z \geqq 0, & \end{aligned}$$

where λ is now defined as an $m \times 1$ vector, and $\odot$ denotes the Hadamard product, i.e. the element-by-element multiplication. Each component of this vector λ is able to scale down a corresponding component of the input vector of the observation being evaluated x^0 (see also Färe, Grosskopf and Lovell [1985], pp. 160-162). It must be stressed that, in contrast to the radial approach, the inequality constraints on the components of the efficiency measure must be absolutely incorporated, if not projections can be made on hypersurfaces which are no part of the boundary of the input correspondence.

The calculation of the Zieschang efficiency measure in the inputs $Z_i(x, y)$ requires solving a pair of linear programs for each observation (x^0, y^0). The first step is the calculation of the radial efficiency measure in the inputs $DF_i(x, y)$: λ^* is the optimal value. The second step is the calculation of the Färe-Lovell efficiency measure $FL_i(x, y)$ for the modified observation $(\lambda^* x^0, y^0)$. The Zieschang efficiency measure is simply the product of the efficiency measures obtained in both steps $Z_i(x, y) = FL_i(x\, DF_i(x, y), y)\, DF_i(x, y)$.

The asymmetric Färe efficiency measure in the inputs $AF_i(x, y)$ requires solving m linear programs (one for each input dimension) for each observation (x^0, y^0) and taking the minimum of the m calculated partial efficiency measures $AF_i^j(x, y)$. Or formally, these partial measures $AF_i^j(x, y)$ are obtained as the solutions to the following linear program:

$$\begin{aligned}
&\min_{\lambda_h, z_i} \lambda_h \\
&\text{s.t.} \quad \sum_{i=1}^{k} y_{ij} z_i \geqq y_{0j} \quad \text{for } j = 1, \ldots, n \\
&\qquad \sum_{i=1}^{k} x_{i\ell} z_i \leqq x_{0\ell} \quad \text{for } \ell = 1, \ldots, m;\ \ell \neq h \\
&\qquad x_{0h}\lambda_h - \sum_{i=1}^{k} x_{ih} z_i \geqq 0 \\
&\qquad \sum_{i=1}^{k} z_i = 1 \\
&\qquad \lambda_h \geqq 0,\ z_i \geqq 0 \quad \text{for } i = 1, \ldots, k
\end{aligned}$$

and $AF_i(x, y) = \min\{\lambda_1^*, \ldots, \lambda_m^*\}$ determines the asymmetric Färe efficiency measure in the inputs.

APPENDIX B
Additional analysis of diversification in the sample

Table B.1 presents for all 29 efficient observations the values of the inputs and the outputs (rounded numbers). In addition, it reports an index of specialization, which is defined as follows:

$$S = \frac{\min_{\ell} \frac{y_\ell}{\hat{y}_\ell}}{\max_{\ell} \frac{y_\ell}{\hat{y}_\ell}} \qquad \text{for } \ell = 1, \ldots, m$$

where $\hat{y}_\ell$ is the median output for the ℓ-th dimension in the sample. Clearly, $0 \leqslant S \leqslant 1$. Note that a zero value implies a complete specialization in at least one of the outputs, while a value of unity indicates that an observation is equally diversified as the median of each output dimension in the sample. Finally, descriptive statistics are reported on the same information for the inefficient observations only.

This table can be interpreted as follows. First, for the inefficient observations it is clear that they are on average less specialized and thus more diversified than the efficient observations which span the hyperplanes of the convex

Table B.1: Outputs, inputs and an index of specialization of US banks

Obs	Y1	Y2	Y3	Y4	Y5	X1	X2	X3	S
Efficient observations (29 observations)									
1	37812	48236	2923	13874	1598	299	92902	1001416	0.3517
2	38280	80900	3250	30500	8800	265	485185	419629	0.5129
3	139733	12532	803	1248	646	48	373152	467218	0.0219
4	7455	17000	829	3704	593	58	23793	230546	0.5013
5	1489	6484	405	791	175	15	48451	58258	0.3028
6	13574	26066	1325	7108	2343	103	15452	646172	0.6962
7	151029	104147	3069	52064	9041	876	4413101	4205357	0.2413
8	131614	126655	1874	69793	14820	1166	6706439	11550379	0.1303
9	8916	11688	583	2048	958	66	164174	86963	0.5621
10	17849	118488	773	9570	5538	185	380889	784920	0.1382
11	11411	5678	570385	987	564	71	367587	1310195	0.0004
12	20184	61439	2354	5131	4099	144	221287	420021	0.3646
13	14524	16891	1096	4774	1253	85	8017	386396	0.6519
14	4119	12424	1202	3403	84515	57	205181	309610	0.0059
15	1221	4402	179	858	291	6	29914	92315	0.4948
16	698	1853	43	122	213	5	18000	55086	0.1684
17	3039	5575	127	454	713	20	2260	113894	0.1873
18	1200	226	41	356	390	9	5804	36806	0.0390
19	1182	16212	0	4515	0	20	12390	347664	0.0000
20	9500	29482	342	10613	603	34	125972	183109	0.1564
21	8647	36569	877	10990	555	33	47802	622515	0.1717
22	4815	25192	0	10016	0	51	68814	307679	0.0000
23	28638	103346	632	16018	0	121	242464	1320286	0.0000
24	3783	34607	484	7561	0	22	98207	164678	0.0000
25	1254	8903	0	3452	0	12	7000	172650	0.0000
26	10714	85061	16145	3183	1	210	73829	1518068	0.0000
27	2213	7562	103	3762	0	8	35638	72101	0.0000
28	36437	404045	0	151828	0	333	1441161	6237352	0.0000
29	4745	40305	4564	6794	0	91	21183	495702	0.0000
Mean	24692	50068	21186	15018	4749	152	542622	1159206	0.1965
Minimum	698	226	0	122	0	5	2260	36806	0.0000
Maximum	151029	404045	570385	151828	84515	1166	6706439	11550379	0.6962
St. Dev.	41015	76550	103832	30052	15457	255	1424837	2353889	0.2218
Inefficient observations (546 observations)									
Mean	11678	24164	1787	5468	1303	109	532642	1028300	0.2948
Minimum	136	392	0	0	0	5	8448	45111	0.0000
Maximum	95777	194116	29886	74703	11803	914	7608838	9536301	0.8676
St. Dev.	12871	29823	3640	7738	1448	119	741603	1300030	0.2036

DEA hull. Furthermore, 52 of these observations are completely specialized in at least a single dimension $(S = 0)$. Second, for the efficient observations one finds, as stated earlier, 8 observations with a complete specialization $(S = 0)$. Moreover, 20 out of 29 observations have an index of specialization below the overall sample average of .2898.

These observations are consistent with our hypothesis that the large variation in the degree of specialization and diversification cause the observed pattern of technical efficiency scores. This is also confirmed in a simple analysis of correlation between the four technical efficiency measures and the above defined index of specialization. The product-moment correlation coefficients for the radial, the Färe-Lovell, the Zieschang and the asymmetric Färe efficiency measures are respectively $-.233$, $-.212$, $-.204$ and $-.179$. They all have the expected negative sign and differ significantly from zero.

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