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Plant capacity notions: review, new definitions, and existence results at firm and industry levels

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ABSTRACT

This study investigates the existence of solutions for the key plant capacity utilisation (PCU) concepts using general nonparametric technologies. This is done via a theoretical review of existing and some new PCU concepts. Focusing on short-run and long-run output-oriented, attainable output-oriented, and input-oriented PCU notions, we first investigate the existence of solutions at the firm level. Under mild axioms, this question regarding the existence of solutions for these PCU concepts at the firm level is affirmatively answered under variable and constant returns to scale as well as under convex and nonconvex assumptions. However, short-run and long-run output-oriented and attainable output-oriented PCU concepts may not be implementable depending on certain conditions. There are no such reservations for the input-oriented PCU. Then, for this same range of PCU concepts, we explore the more difficult question as to the existence of solutions at the industry level. The output-oriented and attainable output-oriented PCU exist at the industry level under strict conditions: existence and attainability are interwoven at this level. The industry input-oriented PCU is always feasible at the industry model. This theoretical review is supplemented by a semi-systematic empirical review, and an empirical application. We conclude that input-oriented PCU is clearly the best concept.

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1. Introduction

Johansen (1968) is probably the first to introduce a technical or engineering approach to capacity measurement by proposing the plant capacity concept in the economic literature using single output production functions. In particular, he informally defines plant capacity by the maximal amount of output that can be produced per unit of time with existing plants and equipment without restrictions on the amount of available variable inputs. On the one hand, Färe (1988) (hence F88) and Färe, Grosskopf, and Kokkelenberg (1989) (hence FGK89) and on the other hand Färe, Grosskopf, and Valdmanis (1989) (hence FGV89) provide an operational way to measure this output-oriented (O-oriented) plant capacity notion using a nonparametric frontier framework focusing on a single output and multiple outputs, respectively. Using a general specification of a nonparametric frontier technology (e.g. F88), plant capacity utilisation can then be determined from observed input and output data by calculating a couple of O-oriented efficiency measures. This O-oriented plant capacity has been applied in a series of empirical applications mainly in health care (e.g. Kerr et al. 1999) and in fisheries (e.g. Vestergaard, Squires, and Kirkley 2003). We are also aware of one empirical application in farming (e.g. Liu et al. 2019) and another macroeconomic application on trade barriers (e.g. Badau 2015). Empirical applications in sectors like construction, manufacturing, public bus companies, steel and iron firms, and universities are also available, among others (see Table 3 infra). Fukuyama et al. (2021) summarise some recent attempts to extend the O-oriented plant capacity notion to include the expansion of good outputs and the reduction of bad outputs using a more general efficiency measure and apply it to the iron and steel industry. Zhang et al. (2020) are another example of such a modelling strategy focusing on transportation in 30 Chinese provinces and cities over the period 2011–2017. These authors find some capacity utilisation variation over time and report significant regional differences.

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Alternatively, Cesaroni, Kerstens, and Van de Woestyne (2017) (hence CKVDW17) adopt the same nonparametric frontier framework to propose a new input-oriented (I-oriented) measure of plant capacity utilisation based on a pair of variable I-oriented efficiency measures. Their empirical illustration reveals that traditional O-oriented and new I-oriented plant capacity concepts measure different things and lead to different rankings. Complementary to this I-oriented plant capacity notion based on variable inputs, we define a new I-oriented plant capacity concept based on efficiency measures focusing on the fixed inputs solely.

Kerstens, Sadeghi, and Van de Woestyne (2019b) (hence KSVDW19b) argue and empirically illustrate that the above notion of O-oriented plant capacity is unrealistic in that the amounts of variable inputs needed to reach the maximum capacity outputs may be unavailable at either the firm or the industry level. This criticism goes back to the so-called attainability issue already described in Johansen (1968). To remedy this problem, KSVDW19b propose a new attainable O-oriented plant capacity notion that bounds the available amount of variable inputs.

Cesaroni, Kerstens, and Van de Woestyne (2019) (hence CKVDW19) define new long-run (LR) O-oriented as well as I-oriented plant capacity concepts: these allow for changes in all input dimensions simultaneously rather than solely allowing for changes in the variable input dimensions. The plant capacity concepts focusing on changes in the variable inputs alone can then be re-interpreted as short-run (SR) plant capacity concepts.

These various SR and LR O-oriented and I-oriented plant capacity measures have been empirically applied to measure hospital capacity in the Hubei province in China during the recent COVID epidemic in Kerstens and Shen (2021). Though the sample is limited, the empirical evidence indicates that the LR I-oriented plant capacity notion correlates best with the observed mortality. This may lead empirical researchers to reconsider their choice of plant capacity concept. Recent empirical applications of these same four plant capacity notions have been reported in Shen, Balezentis, and Streimikis (2022) and Song et al. (2023).

This study sets itself four main objectives. First, all of the above cited articles assume the existence of results for the required efficiency measures within the nonparametric frontier framework (in particular, the methodological articles of F88, FGK89, FGV89, CKVDW17, KSVDW19b and CKVDW19). Apart from the result in Färe (1984) showing that O-oriented plant capacity cannot be obtained for certain popular parametric specifications of a single output production function (e.g. the CES production function under certain parameter restrictions), no existence results exist for general nonparametric frontier technologies at the firm level. In particular, no such existence results are available for the traditional O-oriented plant capacity: the mere existence of empirical studies computing a certain concept is no substitute for formal existence results delineating the exact conditions under which such empirical results can be obtained. Surely no such existence results are known to us for the new attainable O-oriented and I-oriented plant capacity notions. Furthermore, no existence results are accessible for the new LR plant capacity concepts.

Second, no existence result is known to us at the level of the industry for any of the mentioned plant capacity notions. This is an even bigger issue than existence at the firm level, since it may well be possible that a certain plant capacity concept exists at the firm level but fails to hold at the industry level. For instance, take the traditional O-oriented plant capacity as a case in point. It is regularly computed in the empirical literature and therefore seems to exist. But, given the attainability issue raised by KSVDW19b it may well be that not all firms in an industry are capable to reach their full O-oriented plant capacity simultaneously. From a theoretical and empirical point of view, one may prefer using a plant capacity notion that always exists at both the firm and industry levels.

Third, while the seminal contributions of F88, FGK89 and FGV89 determine plant capacity on constant returns to scale (CRS) technologies, Kerstens and Shen (2021) instead favour the use of variable returns to scale (VRS) technologies and identify four other hospital capacity studies doing similarly. We add a semi-systematic survey of empirical applications (e.g. Snyder 2019) showing, among others, that most studies impose VRS. It should be noted that the SR and LR I-oriented measures of plant capacity utilisation have so far only been defined for VRS technologies. It is an open question whether these SR and LR notions can be defined relative to CRS technologies. We manage to provide a theoretical solution to this problem. Furthermore, it turns out that the LR O-oriented plant capacity under CRS technologies requires a similar solution approach. This potentially enlarges the toolbox for the empirical practitioner.

Fourth, we provide a new definition for an SR I-oriented plant capacity concept focusing on fixed input dimensions and for an LR attainable O-oriented plant capacity notion that were hitherto missing in the literature. Furthermore, under CRS technologies we provide some bounds on the theoretical solutions that we have devised for the SR and LR I-oriented plant capacity notions as well as for the LR O-oriented plant capacity concept.

This rich set of plant capacity concepts and their estimation strategies developed in the economics and operations research literature has – to the best of our knowledge – so far not yet made an inroad in the operations management literature. While capacity and its utilisation are one of the key critical elements in the so-called factory physics framework proposed by Hopp and Spearman (2011), we are unaware of operations management literature making use of these engineering or plant capacity concepts.

This study is structured as follows. Section 2 prepares the floor by defining general technologies, the required nonparametric frontier technologies as well as the necessary efficiency measures. Section 3 defines various SR and LR plant capacity notions and proves their existence at the firm level. This leads to the definition of a new LR attainable O-oriented plant capacity concept. Section 4 verifies whether these same plant capacity concepts also exist at the industry level. This is first done for the SR concepts and we indicate how the results transpose to the LR plant capacity concepts. Section 5 discusses some numerical issues related to the definition of some plant capacity concepts under a constant return to scale assumption. Section 6 starts with a semi-systematic survey of empirical applications summarising some basic characteristics of existing studies (e.g. the majority imposes VRS). It continues with an empirical application. Conclusions wrap up the main results in Section 7.

2. Technology and efficiency measures: definitions

2.1. Technology: definitions and axioms

We start by defining the technology and some basic notation. Given an N-dimensional input vector $\mathbf{x} \in \mathbb{R}^N_+$ and an M-dimensional output vector $\mathbf{y} \in \mathbb{R}^M_+$, the production possibility set or technology T is defined as $T = \{(\mathbf{x}, \mathbf{y}) \mid \mathbf{x}$ can produce at least $\mathbf{y}\}$.[1] Commonly, the following conditions are imposed on input and output data defining the technology (see, e.g. Färe, Grosskopf, and Lovell 1994, 44–45): (D.1) each firm utilises nonnegative amounts of each input to produce nonnegative amounts of each output; (D.2) there exists an aggregate production of positive amounts of every output, and an aggregate use of positive amounts of every input; and (D.3) each firm uses a positive amount of at least one input to produce a positive amount of at least one output.

Associated with this technology T, the input set $L(\mathbf{y}) = \{\mathbf{x} \mid (\mathbf{x}, \mathbf{y}) \in T\}$ contains all input vectors $\mathbf{x}$ that yield at least a given output vector $\mathbf{y}$. Similarly, associated with technology T one can define an output set $P(\mathbf{x}) = \{\mathbf{y} \mid (\mathbf{x}, \mathbf{y}) \in T\}$ that contains all output vectors $\mathbf{y}$ that can be generated from at most a given input vector $\mathbf{x}$.

The technology T, input set $L(\mathbf{y})$, and output set $P(\mathbf{x})$ are related as follows (F88(p. 5)):

$$(\mathbf{x}, \mathbf{y}) \in T \iff \mathbf{x} \in L(\mathbf{y}) \iff \mathbf{y} \in P(\mathbf{x}). \qquad (1)$$

Though input set, output set as well as technology represent the same production technology, each highlights a different aspect. The input set focuses on input substitution, the output set centres on output substitution, and the technology T aims at the transformation of inputs into outputs (F88(p. 5)).

In our contribution, technology T respects some combination of the following axioms:

(T.1) Possibility of inaction and no free lunch, i.e. $(\mathbf{0}, \mathbf{0}) \in T$ and if $(\mathbf{0}, \mathbf{y}) \in T$, then $\mathbf{y} = \mathbf{0}$.

(T.2) T is a closed subset of $\mathbb{R}^N_+ \times \mathbb{R}^M_+$.

(T.3) Strong disposal of inputs and outputs, i.e. if $(\mathbf{x}, \mathbf{y}) \in T$ and $(\mathbf{x}', \mathbf{y}') \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$, then $(\mathbf{x}', -\mathbf{y}') \geq (\mathbf{x}, -\mathbf{y}) \Rightarrow (\mathbf{x}', \mathbf{y}') \in T$.

(T.4) $(\mathbf{x}, \mathbf{y}) \in T \Rightarrow \delta(\mathbf{x}, \mathbf{y}) \in T$ for $\delta \in \Gamma$, where
- (i) (i)$\Gamma \equiv \text{CRS} = \{\delta \mid \delta \geq 0\}$;
- (ii) (ii)$\Gamma \equiv \text{VRS} = \{\delta \mid \delta = 1\}$.

(T.5) T is convex.

These traditional axioms on technology merit the following remarks (see Färe, Grosskopf, and Lovell 1994). Production can be halted (inaction) and without inputs one cannot generate any outputs (no free lunch). The production possibility set is closed. Inputs can be wasted, and outputs can be destroyed at no opportunity costs (strong or free disposability of inputs and outputs). We consider two returns to scale assumptions: either CRS or VRS. Finally, technology is convex. Observe that these axioms are not always maintained in this contribution.[2] Specifically, central axioms distinguishing the technologies in the empirical analysis are: (i) CRS versus VRS and (ii) convexity (C) versus nonconvexity (NC).

In economics it is customary to distinguish in the SR between fixed and variable inputs depending on whether inputs are exogenous to managerial control or are fully controlled by management. This leads to a partitioning of the input vector $\mathbf{x}$ into a fixed ($\mathbf{x}^f$) and variable part ($\mathbf{x}^v$). One can denote $\mathbf{x} = (\mathbf{x}^f, \mathbf{x}^v)$ with $\mathbf{x}^f \in \mathbb{R}^{N_f}_+$ and $\mathbf{x}^v \in \mathbb{R}^{N_v}_+$ such that $N = N_f + N_v$. To simplify, it is assumed that all producers share common subvectors of fixed and variable input dimensions.

Partitioning the input vector requires sharpening the conditions on inputs and outputs. In particular, FGK89 (p. 659–660) state: (D.4) each fixed input is used by some firm, and each firm uses some fixed input. We also need:

(D.5) each variable input is used by some firm, and each firm uses some variable input.

Based on FGV89, we can define an SR technology $T^f = \{(\mathbf{x}^f, \mathbf{y}) \in \mathbb{R}_+^{N_f} \times \mathbb{R}_+^M \mid$ there exist some $\mathbf{x}^v$ such that $(\mathbf{x}^f, \mathbf{x}^v)$ can produce at least $\mathbf{y}\}$ as well as the corresponding output set $P^f(\mathbf{x}^f) = \{\mathbf{y} \mid (\mathbf{x}^f, \mathbf{y}) \in T^f\}$.

2.2. Nonparametric frontier technologies

Consider K observations ($k = 1, \ldots, K$) with each a vector of inputs and outputs $(\mathbf{x}_k, \mathbf{y}_k) \in \mathbb{R}_+^N \times \mathbb{R}_+^M$. The corresponding C and NC nonparametric frontier technologies under the CRS and VRS assumptions, as well as the input and output sets, can be mathematically represented as follows:

$$T_{\Lambda,\Gamma} = \Big\{(\mathbf{x}, \mathbf{y}) \mid \mathbf{x} \geq \sum_{k=1}^K \delta z_k \mathbf{x}_k, \quad \mathbf{y} \leq \sum_{k=1}^K \delta z_k \mathbf{y}_k, \mathbf{z} = (z_1, \ldots, z_K) \in \Lambda, \quad \delta \in \Gamma \Big\}, \tag{2}$$

$$L_{\Lambda,\Gamma}(\mathbf{y}_p) = \{\mathbf{x} \mid \mathbf{x} \geq \sum_{k=1}^K \delta z_k \mathbf{x}_k, \tag{1}$$

$$\mathbf{y}_p \leq \sum_{k=1}^K \delta z_k \mathbf{y}_k, \mathbf{z} \in \Lambda, \delta \in \Gamma \}, \tag{3}$$

$$P_{\Lambda,\Gamma}(\mathbf{x}_p) = \Big\{\mathbf{y} \mid \mathbf{x}_p \geq \sum_{k=1}^K \delta z_k \mathbf{x}_k, \quad \mathbf{y} \leq \sum_{k=1}^K \delta z_k \mathbf{y}_k, \mathbf{z} \in \Lambda, \delta \in \Gamma \Big\}, \tag{4}$$

where $(\mathbf{x}_p, \mathbf{y}_p)$ is the unit under evaluation; Λ is either C or NC as follows:

(i) $\Lambda \equiv C = \{\mathbf{z} \mid \sum_{k=1}^K z_k = 1$ and $\forall k \in \{1, \ldots, K\}: z_k \geq 0\}$;
(ii) $\Lambda \equiv NC = \{\mathbf{z} \mid \sum_{k=1}^K z_k = 1$ and $\forall k \in \{1, \ldots, K\}: z_k \in \{0, 1\}\}$,

and Γ is either CRS or VRS as defined in axiom (T.4). The activity vector z_k allows to take combinations of observations and is used to model either the convexity assumption or its negation (i.e. nonconvexity). The scalar δ is used to allow for no scaling (VRS) or for a ray unbounded scaling (CRS). More details on constructing nonparametric frontier technologies are found in F88.

The SR technology $T^f_{\Lambda,\Gamma}$ can be represented algebraically as follows:

$$T^f_{\Lambda,\Gamma} = \Big\{(\mathbf{x}^f, \mathbf{y}) \mid \mathbf{x}^f \geq \sum_{k=1}^K \delta z_k \mathbf{x}^f_k, \mathbf{x}^v \geq \sum_{k=1}^K \delta z_k \mathbf{x}^v_k, \quad \mathbf{y} \leq \sum_{k=1}^K \delta z_k \mathbf{y}_k, \mathbf{z} \in \Lambda, \delta \in \Gamma \Big\}. \tag{5}$$

The SR output set $P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p)$ is represented algebraically by

$$P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p) = \{\mathbf{y} \mid \mathbf{x}^f_p \geq \sum_{k=1}^K \delta z_k \mathbf{x}^f_k, \mathbf{x}^v \geq \sum_{k=1}^K \delta z_k \mathbf{x}^v_k, \quad \mathbf{y} \leq \sum_{k=1}^K \delta z_k \mathbf{y}_k, \mathbf{z} \in \Lambda, \delta \in \Gamma \}. \tag{6}$$

Proposition 2.1: *The variable input constraints are redundant at the firm level and can be removed from the SR technology $T^f_{\Lambda,\Gamma}$ and from the SR output set $P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p)$ at the firm level.*

The proof of Proposition 2.1 as well as the other statements are available in Appendix A. Based on Proposition 2.1, we can eliminate constraint $\mathbf{x}^v \geq \sum_{k=1}^K \delta z_k \mathbf{x}^v_k$ from (5) and (6): this result simplifies computations, and it is valid for CRS or VRS and for C or NC technologies alike.

Remark 2.1: In the literature, one can find three variations on the definition of the SR technology $T^f_{\Lambda,\Gamma}$ that are compatible with our formulation.

- F88, FGK89 and FGV89 all drop the variable input constraints from their definition of the SR technology (5) and (6). This can only be meaningfully interpreted if the authors implicitly have the above variable input constraints in mind whereby the amount of variable inputs are decision variables ($\mathbf{x}^v$). Only then, these variable input constraints are redundant.
- In Färe, Grosskopf, and Lovell (1994, 262) a related argument contains a minor typo: in our notation, it is argued that $\sum_{k=1}^K \delta z_k \mathbf{x}^v_k = \boldsymbol{\lambda} \mathbf{x}^v_p$ with $\boldsymbol{\lambda} \in \mathbb{R}_+^{N_v}$ and variable inputs as parameters ($\mathbf{x}^v_p$). However, this constraint is not redundant in general, and only $\sum_{k=1}^K \delta z_k \mathbf{x}^v_k \leq \boldsymbol{\lambda} \mathbf{x}^v_p$ can make these variable input constraints redundant.
- In CKVDW19 (p. 388) and Kerstens, Sadeghi, and Van de Woestyne (2019a, 701) the SR technology $T^f_{\Lambda,\Gamma}$ is considered as a projection of the general technology $T_{\Lambda,\Gamma}$ into the subspace of fixed inputs and outputs, i.e.

technology $T^f_{\Lambda,\Gamma}$ is obtained by a projection of technology $T_{\Lambda,\Gamma} \in \mathbb{R}^{N+M}_+$ into the subspace $\mathbb{R}^{N_f+M}_+$ (i.e. by setting all variable inputs equal to zero). By analogy, the same applies to the output set $P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p)$ (it is straightforward with Proposition 2.1). Note that by fixing all variable inputs to any identical numerical value one again makes the variable input constraints redundant.

Note that the input set $L_{\Lambda,\Gamma}(\mathbf{y}_p)$ and the output set $P_{\Lambda,\Gamma}(\mathbf{x}_p)$ are nonempty and closed sets. Also, the output set $P_{\Lambda,\Gamma}(\mathbf{x}_p)$ is a bounded set. This guarantees the existence of I- and O-oriented efficiency measures (see Section 2.3). In Theorem 2.1, we prove that the SR output set $P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p)$ is a nonempty and compact set.

Theorem 2.1: *The SR output set $P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p)$ is a nonempty and compact set.*

Thus the SR output set $P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p)$ is nonempty and compact under the C and NC assumptions as well as in the CRS and VRS cases. Therefore, Theorem 2.1 guarantees the existence of the SR O-oriented efficiency measure (see Section 2.3).

Generalising CKVDW17 (p. 727), one can define the following atypical definition: $L_{\Lambda,\Gamma}(\mathbf{0}) = \{\mathbf{x} \mid (\mathbf{x},\mathbf{0}) \in T_{\Lambda,\Gamma}\}$ is the input set compatible with a zero output level. This input set indicates the input levels where non-zero production is initiated. The input set $L_{\Lambda,\Gamma}(\mathbf{0})$ can be obtained by (3) when we replace the output constraint $\mathbf{y}_p \leq \sum_{k=1}^K \delta z_k \mathbf{y}_k$ with $\mathbf{0} \leq \sum_{k=1}^K \delta z_k \mathbf{y}_k$.

Proposition 2.2: *The output constraints are redundant at the firm level and can be removed from the SR input set $L_{\Lambda,\Gamma}(\mathbf{0})$ at the firm level.*

Proposition 2.2 simplifies computations, and it is valid for CRS or VRS and for C or NC technologies as well.

We introduce $L_{\Lambda,\Gamma}(\mathbf{y}_{\min}) = \{\mathbf{x} \mid (\mathbf{x},\mathbf{y}_{\min}) \in T_{\Lambda,\Gamma}\}$, whereby $\mathbf{y}_{\min} = \min_{k=1,\dots,K} \mathbf{y}_k$. Therefore, the minimum output is determined component-wise for every output $\mathbf{y}$ over all units K under both the C and NC cases and for the CRS and VRS axioms. Moreover, let $L_{\Lambda,\Gamma}(\mathbf{y}^\epsilon) = \{\mathbf{x} \mid (\mathbf{x},\mathbf{y}^\epsilon) \in T_{\Lambda,\Gamma}\}$ where $\mathbf{y}^\epsilon \in \mathbb{R}^M_+$ is a vector with arbitrary small components and $\mathbf{y}^\epsilon \leq \mathbf{y}_{\min}$: this inequality is compatible with the assumption of strong output disposal. Note that $L_{\Lambda,\Gamma}(\mathbf{y}^\epsilon) = \{\mathbf{x} \mid (\mathbf{x},\mathbf{y}^\epsilon) \in T_{\Lambda,\Gamma}\}$ is the input set compatible with a $\mathbf{y}^\epsilon$ output level. This input set denotes the input levels where production is started up. Note that $L_{\Lambda,\Gamma}(\mathbf{0})$, $L_{\Lambda,\Gamma}(\mathbf{y}_{\min})$ and $L_{\Lambda,\Gamma}(\mathbf{y}^\epsilon)$ are nonempty and closed sets.

Proposition 2.3: (i) *Under VRS, we have $L_{\Lambda,VRS}(\mathbf{0}) = L_{\Lambda,VRS}(\mathbf{y}^\epsilon) = L_{\Lambda,VRS}(\mathbf{y}_{\min}) \subset \mathbb{R}^N_+$.*
(ii) *Under CRS, we have $L_{\Lambda,CRS}(\mathbf{y}_{\min}) \subseteq L_{\Lambda,CRS}(\mathbf{y}^\epsilon) \subset L_{\Lambda,CRS}(\mathbf{0}) = \mathbb{R}^N_+$.*

Under the VRS assumption, for each output level $\mathbf{y} \leq \mathbf{y}_{\min}$ we have the same input set $L_{\Lambda,VRS}(\mathbf{y})$. While under the CRS assumption, a higher output level ($\mathbf{0} \leq \mathbf{y}^\epsilon \leq \mathbf{y}_{\min}$) leads to a smaller input set ($L_{\Lambda,CRS}(\mathbf{y}_{\min}) \subseteq L_{\Lambda,CRS}(\mathbf{y}^\epsilon) \subset L_{\Lambda,CRS}(\mathbf{0})$). Moreover, under the VRS case we have $L_{\Lambda,VRS}(\mathbf{0}) \subset \mathbb{R}^N_+$ while under the CRS case we have $L_{\Lambda,CRS}(\mathbf{0}) = \mathbb{R}^N_+$. Proposition 2.3 is discussed in detail in Figure 1(b) infra and we show that how the value of $\mathbf{y}^\epsilon$ determines the quality of the solutions for the CRS case.

Extending CKVDW19, we now define the particular output set $P_{\Lambda,\Gamma} = \{\mathbf{y} \mid \exists\, \mathbf{x} : (\mathbf{x},\mathbf{y}) \in T_{\Lambda,\Gamma}\}$ including all possible outputs irrespective of the needed inputs. The LR output set $P_{\Lambda,\Gamma}$ is represented algebraically by

$$P_{\Lambda,\Gamma} = \Big\{\mathbf{y} \;\Big|\; \mathbf{x} \geq \sum_{k=1}^K \delta z_k \mathbf{x}_k, \quad \mathbf{y} \leq \sum_{k=1}^K \delta z_k \mathbf{y}_k, \mathbf{z} \in \Lambda, \delta \in \Gamma \;\Big\}. \quad (7)$$

Note that $P_{\Lambda,\Gamma}$ is a non-empty and closed set under both VRS and CRS cases.

Let $P^{\mathbf{x}_{\max}}_{\Lambda,\Gamma} = \{\mathbf{y} \mid \exists \mathbf{x} : \mathbf{x} \leq \mathbf{x}_{\max};\ (\mathbf{x},\mathbf{y}) \in T_{\Lambda,\Gamma}\}$, whereby $\mathbf{x}_{\max} = \max_{k=1,\dots,K} \mathbf{x}_k$. Hence, the maximum input is taken on each component for every input $\mathbf{x}$ over all observed units K under both the C and NC cases and the CRS and VRS assumptions.

Moreover, let $P^{\mathbf{x}^\epsilon}_{\Lambda,\Gamma} = \{y \mid \exists \mathbf{x} : \mathbf{x} \leq \mathbf{x}^\epsilon;\ (\mathbf{x},\mathbf{y}) \in T_{\Lambda,\Gamma}\}$ where $\mathbf{x}^\epsilon \in \mathbb{R}^M_+$ is a vector with an arbitrary components such that $\mathbf{x}^\epsilon \geq \mathbf{x}_{\max}$. Note that the inequality $\mathbf{x}^\epsilon \geq \mathbf{x}_{\max}$ is justified by the assumption of strong disposal of the inputs.

Then, we have the following Proposition 2.4:

Proposition 2.4: (i) *Under VRS, we have $P_{\Lambda,VRS} = P^{\mathbf{x}_{\max}}_{\Lambda,VRS} = P^{\mathbf{x}^\epsilon}_{\Lambda,VRS} \subset \mathbb{R}^M_+$.*
(ii) *Under CRS, we have $P^{\mathbf{x}_{\max}}_{\Lambda,CRS} \subseteq P^{\mathbf{x}^\epsilon}_{\Lambda,CRS} \subset P_{\Lambda,CRS} = \mathbb{R}^M_+$.*

Under the VRS assumption, for each upper input level $\mathbf{x} \geq \mathbf{x}_{\max}$ we have the same LR output set $P_{\Lambda,VRS}$. While under the CRS assumption, a higher upper input level leads to a larger LR output set. Proposition 2.4 is illustrated in detail in Figure 2 infra and we show how the value of $\mathbf{x}^\epsilon$ determines the quality of the solutions for the CRS case.

Based on Proposition 2.4(i), $P_{\Lambda,\Gamma}$ when $\Lambda = \{C, NC\}$ and $\Gamma = VRS$ can be equivalently defined by $P^{\mathbf{x}^\epsilon}_{\Lambda,VRS}$, whereby $\mathbf{x}^\epsilon \geq \mathbf{x}_{\max}$. Moreover, $P_{\Lambda,\Gamma}$ is a bounded set under the VRS case, but not under the CRS case. In fact, under the CRS case we have $P_{\Lambda,CRS} = \mathbb{R}^M_+$, while $P^{\mathbf{x}_{\max}}_{\Lambda,CRS} \subseteq P^{\mathbf{x}^\epsilon}_{\Lambda,CRS} \subset \mathbb{R}^M_+$. Note that $P^{\mathbf{x}^\epsilon}_{\Lambda,\Gamma}$ and $P^{\mathbf{x}_{\min}}_{\Lambda,\Gamma}$ are nonempty, closed and bounded sets.

2.3. Efficiency measures

The radial output efficiency measure characterises the output set $P_{\Lambda,\Gamma}(\mathbf{x})$ completely and can be defined as follows:

$$DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma) = \max\{\varphi \mid \varphi \geq 0, \varphi \mathbf{y}_p \in P_{\Lambda,\Gamma}(\mathbf{x}_p)\}. \quad (8)$$

It is larger than or equal to unity ($DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma) \geq 1$), with efficient production on the boundary (isoquant) of the output set $P_{\Lambda,\Gamma}(\mathbf{x}_p)$ represented by unity, and it happens to have a revenue interpretation (e.g. Färe, Grosskopf, and Lovell 1994).

Next, we define the efficiency measure $DF_o(\mathbf{y}_p \mid P_{\Lambda,\Gamma})$ that does not depend on a particular input vector $\mathbf{x}_p$:

$$DF_o(\mathbf{y}_p \mid P_{\Lambda,\Gamma}) = \max\{\varphi \mid \varphi \geq 0, \varphi \mathbf{y}_p \in P_{\Lambda,\Gamma}\}. \quad (9)$$

Contrary to the radial output efficiency measure (8), this efficiency measure $DF_o(\mathbf{y}_p \mid P_{\Lambda,\Gamma})$ is allowed to choose the inputs needed for maximising φ. Clarifications for this peculiar concept can be found in Figures 1–3 in CKVDW19 (p. 390 and 393).

Proposition 2.5: *$DF_o(\mathbf{y}_p \mid P_{\Lambda,\Gamma})$ exists under the VRS assumption, but it does not exist under the CRS case.*

The next proposition illustrates the relation among the values of $DF_o(\mathbf{y}_p \mid P_{\Lambda,\Gamma})$, $DF_o(\mathbf{y}_p \mid P^{\mathbf{x}_{\max}}_{\Lambda,\Gamma})$ and $DF_o(\mathbf{y}_p \mid P^{\mathbf{x}^\epsilon}_{\Lambda,\Gamma})$ when $\Gamma = VRS$ and $\Gamma = CRS$, respectively.

Proposition 2.6: *We have:*

(i) $DF_o(\mathbf{y}_p \mid P_{\Lambda,VRS}) = DF_o(\mathbf{y}_p \mid P^{\mathbf{x}_{\max}}_{\Lambda,VRS}) = DF_o(\mathbf{y}_p \mid P^{x^\epsilon}_{\Lambda,VRS})$;

(ii) $DF_o(\mathbf{y}_p \mid P^{\mathbf{x}_{\max}}_{\Lambda,CRS}) \leq DF_o(\mathbf{y}_p \mid P^{x^\epsilon}_{\Lambda,CRS}) < DF_o(\mathbf{y}_p \mid P_{\Lambda,CRS})$.

Under the VRS assumption, for each input level $\mathbf{x} \geq \mathbf{x}_{\max}$ we have exactly the same LR output efficiency measure $DF_o(\mathbf{y}_p \mid P_{\Lambda,VRS})$. While under the CRS assumption, higher input bounds lead to a bigger LR O-oriented efficiency measure, with an ∞ efficiency measure for $P_{\Lambda,CRS}$. Therefore, the LR O-oriented efficiency measure $DF_o(\mathbf{y}_p \mid P_{\Lambda,VRS})$ can be equivalently formulated as $DF_o(\mathbf{y}_p \mid P^{\mathbf{x}^\epsilon}_{\Lambda,VRS})$. We define the LR O-oriented efficiency measure under both VRS and CRS cases as follows:

$$DF_o(\mathbf{y}_p \mid P^{\mathbf{x}^\epsilon}_{\Lambda,\Gamma}) = \max\{\varphi \mid \varphi \geq 0, \varphi \mathbf{y}_p \in P^{\mathbf{x}^\epsilon}_{\Lambda,\Gamma}\}. \quad (10)$$

Based on Propositions 2.5 and 2.6, $DF_o(\mathbf{y}_p \mid P^{\mathbf{x}^\epsilon}_{\Lambda,\Gamma}) < \infty$ under both CRS and VRS cases.

Denoting the radial output efficiency measure of the SR output set $P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p)$ by $DF^f_o(\mathbf{x}^f_p, \mathbf{y}_p \mid \Lambda, \Gamma)$, this short-run O-oriented efficiency measure is defined in the following way:

$$DF^f_o(\mathbf{x}_p{}^f, \mathbf{y}_p \mid \Lambda, \Gamma) = \max\{\varphi \mid \varphi \geq 0, \varphi \mathbf{y}_p \in P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p)\}. \quad (11)$$

Corollary 2.1: *Note that based on Theorem 2.1, since $P^f_{\Lambda,\Gamma}(\mathbf{x}^f_p)$ is a compact set, then this SR O-oriented efficiency measure $DF^f_o(\mathbf{x}^f_p, \mathbf{y}_p \mid \Lambda, \Gamma)$ always exists.*

Corollary 2.1 is valid for CRS or VRS and for C or NC technologies as well.

The radial input efficiency measure completely characterises the input set $L_{\Lambda,\Gamma}(\mathbf{y}_p)$ and can be defined as follows:

$$DF_i(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma) = \min\{\theta \mid \theta \geq 0, \theta \mathbf{x}_p \in L_{\Lambda,\Gamma}(\mathbf{y}_p)\}. \quad (12)$$

It is smaller than or equal to unity ($DF_i(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma) \leq 1$), with efficient production on the boundary (isoquant) of $L_{\Lambda,\Gamma}(\mathbf{y}_p)$ represented by unity, and it has a cost interpretation (see, e.g. Färe, Grosskopf, and Lovell 1994).

When only reducing the variable inputs, a sub-vector input efficiency measure $DF^{SR}_{vi}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}_p \mid \Lambda, \Gamma)$ is defined as follows:

$$DF^{SR}_{vi}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}_p \mid \Lambda, \Gamma) = \min\{\theta \mid \theta \geq 0, (\mathbf{x}^f_p, \theta \mathbf{x}^v_p) \in L_{\Lambda,\Gamma}(\mathbf{y}_p)\}. \quad (13)$$

When only reducing the fixed inputs, a sub-vector input efficiency measure $DF^{SR}_{fi}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}_p \mid \Lambda, \Gamma)$ is defined as follows:

$$DF^{SR}_{fi}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}_p \mid \Lambda, \Gamma) = \min\{\theta \mid \theta \geq 0, (\theta \mathbf{x}^f_p, \mathbf{x}^v_p) \in L_{\Lambda,\Gamma}(\mathbf{y}_p)\}. \quad (14)$$

The corresponding model of the I-oriented efficiency measures (13) and (14) is feasible and we have $0 < DF^{SR}_{vi}(\mathbf{x}^f, \mathbf{x}^v, \mathbf{y} \mid \Lambda, \Gamma) \leq 1$ and $0 < DF^{SR}_{fi}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}_p \mid \Lambda, \Gamma) \leq 1$.

Reducing all inputs, an I-oriented efficiency measure $DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, \Gamma)$ relative to the input set with zero output level is given by

$$DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, \Gamma) = \min\{\theta \mid \theta \geq 0, \theta \mathbf{x}_p \in L_{\Lambda,\Gamma}(\mathbf{0})\}. \quad (15)$$

Reducing variable inputs only, a sub-vector input efficiency measure $DF^{SR}_{vi}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, \Gamma)$ evaluated relative to the input set with a zero output level is defined as follows:

$$DF^{SR}_{vi}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, \Gamma) = \min\{\theta \mid \theta \geq 0, (\mathbf{x}^f_p, \theta \mathbf{x}^v_p) \in L_{\Lambda,\Gamma}(\mathbf{0})\}. \quad (16)$$

This variable inputs sub-vector efficiency measure is defined with respect to the input set with zero output level where production is initiated.

Reducing fixed inputs only, a sub-vector input efficiency measure $DF^{SR}_{fi}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, \Gamma)$ evaluated relative to the input set with a zero output level is defined as follows:

$$DF^{SR}_{fi}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, \Gamma) = \min\{\theta \mid \theta \geq 0, (\theta \mathbf{x}^f_p, \mathbf{x}^v_p) \in L_{\Lambda,\Gamma}(\mathbf{0})\}. \quad (17)$$

This fixed inputs sub-vector efficiency measure is defined with respect to the input set with zero output level where production is initiated.

The following proposition shows that $DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, VRS)$ and $DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, VRS)$ with $\iota = \{vi, fi\}$ are smaller or equal to unity under VRS and zero in the CRS case.

Proposition 2.7: *We have:*

(i) $0 < DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, VRS) \leq 1$ *and* $0 < DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, VRS) \leq 1$ *with* $\iota = \{vi, fi\}$.

(ii) $DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, CRS) = DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, CRS) = 0$ *with* $\iota = \{vi, fi\}$.

The LR and SR I-oriented efficiency measures (15) and (16) are feasible under CRS and VRS, and we have $0 < DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, VRS) \leq 1$ and $0 < DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, VRS) \leq 1$. But, they are equal to zero under CRS. Proposition 2.7 is illustrated in detail in Figure 1(a).

The next proposition illustrates the relation among the values of $DF_i(\mathbf{x}_p, \mathbf{y} \mid \Lambda, \Gamma)$ when $\mathbf{y} = \mathbf{0}, \mathbf{y}^\epsilon$ and $\mathbf{y}_{\min}$ respectively.

Proposition 2.8: *We have:*

(i) $DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, VRS) = DF_i(\mathbf{x}_p, \mathbf{y}^\epsilon \mid \Lambda, VRS) = DF_i(\mathbf{x}_p, \mathbf{y}_{\min} \mid \Lambda, VRS)$;

(i) $DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, CRS) < DF_i(\mathbf{x}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS) < DF_i(\mathbf{x}_p, \mathbf{y}_{\min} \mid \Lambda, CRS)$.

Proposition 2.8 is illustrated in detail in Figure 1(a) infra. Under the VRS assumption, for each output level $\mathbf{y} \leq \mathbf{y}_{\min}$ we have exactly the same input efficiency measure $DF_i(\mathbf{x}_p, \mathbf{y}_{\min} \mid \Lambda, VRS)$. While under the CRS assumption, higher output levels lead to a bigger LR input efficiency measure implying higher efficiency levels. Therefore, the LR input efficiency measure $DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, VRS)$ can be equivalently formulated as either $DF_i(\mathbf{x}_p, \mathbf{y}_{\min} \mid \Lambda, VRS)$ or $DF_i(\mathbf{x}_p, \mathbf{y}^\epsilon \mid \Lambda, VRS)$. We define the LR input efficiency measure under both VRS and CRS cases as follows:

$$DF_i(\mathbf{x}_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma) = \min\{\theta \mid \theta \geq 0, \theta \mathbf{x}_p \in L_{\Lambda,\Gamma}(\mathbf{y}^\epsilon)\}. \quad (18)$$

The following proposition illustrates the relation among the values of $DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y} \mid \Lambda, \Gamma)$ with $\iota = \{vi, fi\}$ when $\mathbf{y} = \mathbf{0}, \mathbf{y}^\epsilon$ and $\mathbf{y}_{\min}$ respectively.

Proposition 2.9: *We have:*

(i) $DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, VRS) = DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}^\epsilon \mid \Lambda, VRS) = DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}_{\min} \mid \Lambda, VRS)$;

(ii) $DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{0} \mid \Lambda, CRS) < DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}^\epsilon \mid \Lambda, CRS) \leq DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}_{\min} \mid \Lambda, CRS)$.

Under the VRS assumption, for each output level $\mathbf{y} \leq \mathbf{y}_{\min}$ we have exactly the same SR input efficiency measure $DF^{SR}_{\iota}(\mathbf{x}^f, \mathbf{x}^v, \mathbf{y}_{\min} \mid \Lambda, VRS)$. While under the CRS assumption, higher output levels lead to a bigger input efficiency measure implying higher efficiency levels. Therefore, this sub-vector input efficiency measure $DF^{SR}_{\iota}(\mathbf{x}^f, \mathbf{x}^v, \mathbf{0} \mid \Lambda, VRS)$ is formulated equivalently as either $DF^{SR}_{\iota}(\mathbf{x}^f, \mathbf{x}^v, \mathbf{y}_{\min} \mid \Lambda, VRS)$ or $DF^{SR}_{\iota}(\mathbf{x}^f, \mathbf{x}^v, \mathbf{y}^\epsilon \mid \Lambda, VRS)$. We define the SR input efficiency measure under both VRS and CRS cases as follows:

$$DF^{SR}_{\iota}(\mathbf{x}^f_p, \mathbf{x}^v_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma) = \min\{\theta \mid \theta \geq 0, (\mathbf{x}^f_p, \theta \mathbf{x}^v_p) \in L_{\Lambda,\Gamma}(\mathbf{y}^\epsilon)\}. \quad (19)$$

3. Plant capacity concepts at the firm level

3.1. Short-run plant capacity concepts

Recalling the informal definition by Johansen (1968, 362) as 'the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted', this O-oriented plant capacity notion is made operational by F88, FGK89 and FGV89 using a couple of

O-oriented efficiency measures. We now recall the formal definition of this O-oriented plant capacity utilisation (hence PCU).

Definition 3.1: SR O-oriented PCU_o^{SR} is defined as follows:

$$PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = \frac{DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)}{DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)},$$

where $DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)$ and $DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ are output efficiency measures including, respectively excluding, the variable inputs as defined before in (8) and (11).

Since $1 \le DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma) \le DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$, notice that $0 < PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) \le 1$. Thus SR O-oriented PCU has an upper limit of unity. This leads to the following remark.

Remark 3.1: Note that F88 (p. 70) shows that if we have an upper bound on the fixed inputs, then the SR O-oriented plant capacity $PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, CRS)$ exists at the firm level under CRS and a single output. Therefore, constraints $\mathbf{x}_p^f \ge \sum_{k=1}^{K} \delta z_k \mathbf{x}_k^f$ of fixed inputs in (6) guarantee that the SR O-oriented efficiency measure $DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, CRS)$ exists and therefore $PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, CRS)$ also exists. If we do not have any fixed inputs, i.e. all inputs are variable (in case that data property (D.4) is not respected by the data), then there is no guarantee that $PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, CRS)$ exists under CRS (see also the LR O-oriented plant capacity notion that is addressed in Section 3.2). As a result, the SR O-oriented plant capacity $PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ exists at the firm level under both the VRS and CRS cases as well as under both the C and NC assumptions.

Depending on whether one disregards inefficiency or accommodates for the eventual existence of inefficiency, FGK89 distinguish between a so-called biased and an unbiased plant capacity measure $DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ and $PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$, respectively. The latter unbiased plant capacity measures $PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ as a ratio of efficiency measures yields a cleaned notion of O-oriented PCU by removing any existing inefficiency. This O-oriented PCU compares the maximum value of outputs at the level of the current inputs to the maximum value of outputs when unlimited amounts of variable inputs are potentially available. Therefore, it determines how the maximal amount of efficient outputs is connected to the current amount of efficient outputs.

KSVDW19b recently argue and empirically illustrate that this O-oriented $PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ is unrealistic in that the variable input amounts required to reach the maximum capacity outputs may simply be unavailable at either the firm or the industry level. This relates to what Johansen (1968) calls the attainability issue. In management the well-known theory of constraints highlights the ubiquity of at least one constraint conditioning the achievement of organisational goals: this provides an alternative motivation for the attainability issue. Therefore, KSVDW19b defines at the firm level a new attainable O-oriented PCU as follows:

Definition 3.2: SR attainable O-oriented $APCU_o^{SR}$ at attainability level $\bar{\lambda} \in \mathbb{R}_+$ is defined by

$$APCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) = \frac{DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)}{ADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)},$$

where the attainable O-oriented efficiency measure $ADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)$ at a certain attainability level $\bar{\lambda} \in \mathbb{R}_+$ is defined by

$$\begin{aligned} &ADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) \\ &= \max\{\varphi \mid \varphi \ge 0, 0 \le \theta \le \bar{\lambda}, \varphi \mathbf{y}_p \in P^f_{\Lambda,\Gamma}(\mathbf{x}_p^f, \theta \mathbf{x}_p^v)\}. \end{aligned} \tag{20}$$

Again, for $\bar{\lambda} \ge 1$, since $1 \le DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma) \le ADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)$, note that $0 < APCU_o(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) \le 1$. Also, for $\bar{\lambda} < 1$, since $1 \le ADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) \le DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)$, note that $1 \le APCU_o(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)$. Moreover, in this case based on Theorem 2.1 we have $APCU_o(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) < \infty$. As a result, we have the following Corollary.

Corollary 3.1: *The SR attainable O-oriented $APCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)$ exists at the firm level under both the VRS and CRS cases as well as under both the C and NC assumptions.*

Moreover, the same authors remark that when experts cannot determine a plausible value for $\bar{\lambda}$, then one can opt for the I-oriented PCU below that is spared from this attainability issue. Based on the attainable O-oriented PCU, one compares the maximal outputs at the level of observed inputs with the maximal outputs when variable inputs are scaled by $\bar{\lambda}$. Therefore, it clarifies how the current value of efficient outputs is connected to the maximal possible values of efficient outputs conditioned by the $\bar{\lambda}$ scalar.

CKVDW17 introduces a variable I-oriented PCU under the VRS assumption using a couple of variable I-oriented efficiency measures.

Definition 3.3: SR VRS variable I-oriented PCU (PCU_{vi}^{SR}) is defined as follows:

$$PCU_{vi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, VRS) = \frac{DF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p \mid \Lambda, VRS)}{DF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{0} \mid \Lambda, VRS)}, \quad (21)$$

where $DF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p \mid \Lambda, VRS)$ and $DF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{0} \mid \Lambda, VRS)$ are the sub-vector variable input efficiency measures defined in (13) and (16), respectively.

We can now define a new fixed I-oriented PCU under the VRS assumption using a couple of fixed I-oriented efficiency measures.

Definition 3.4: SR VRS fixed I-oriented PCU (PCU_{fi}^{SR}) is defined as follows:

$$PCU_{fi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, VRS) = \frac{DF_{fi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p \mid \Lambda, VRS)}{DF_{fi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{0} \mid \Lambda, VRS)}, \quad (22)$$

where $DF_{fi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p \mid \Lambda, VRS)$ and $DF_{fi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{0} \mid \Lambda, VRS)$ are the sub-vector variable input efficiency measures defined in (14) and (17), respectively.

Its interpretation is similar to the variable I-oriented PCU notion in Definition (3.3). It is larger than or equal to unity and it compares the minimum amount of fixed inputs for given amounts of variable inputs and outputs with the minimum amount of fixed inputs with given amounts of variable inputs and output levels where production is initiated. It answers the question how the amount of fixed inputs compatible with the initialisation of production must be scaled up to produce the current amount of outputs. The composing fixed input efficiency measures reveal the amount of over-investment in fixed inputs.

Proposition 3.1: *The SR variable and fixed I-oriented PCUs* (21) *and* (22) *always exist at the firm level under VRS and under both the C and NC assumptions.*

Note that based on Proposition 2.7, we have $DF_{\iota}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{0} \mid \Lambda, CRS) = 0$ with $\iota = \{vi, fi\}$. Hence, Definition 3.3 is invalid under the CRS case. Following CKVDW17, we define an I-oriented PCU notion under CRS using a couple of I-oriented efficiency measures as follows:

Definition 3.5: SR CRS I-oriented PCU (PCU_{ι}^{SR}) can be defined as follows:

$$PCU_{\iota}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \mathbf{y}^{\epsilon} \mid \Lambda, CRS) = \frac{DF_{\iota}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p \mid \Lambda, CRS)}{DF_{\iota}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^{\epsilon} \mid \Lambda, CRS)}, \quad (23)$$

where $DF_{\iota}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p \mid \Lambda, CRS)$ and $DF_{\iota}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^{\epsilon} \mid \Lambda, CRS)$ are the sub-vector input efficiency measures at the current observed output level and at the $\mathbf{y}^{\epsilon}$ level, respectively.

Proposition 3.2: *The SR I-oriented PCU in Definition* 3.5 *always exists at the firm level under CRS and under both C and NC assumptions.*

Note that based on Proposition 2.9, we have $DF_{\iota}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{0} \mid \Lambda, VRS) = DF_{\iota}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^{\epsilon} \mid \Lambda, VRS)$. Hence, a more general definition of the SR I-oriented PCU which is valid under both VRS and CRS cases can be defined as follows:

$$PCU_{\iota}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \mathbf{y}^{\epsilon} \mid \Lambda, \Gamma) = \frac{DF_{\iota}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p \mid \Lambda, \Gamma)}{DF_{\iota}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^{\epsilon} \mid \Lambda, \Gamma)}. \quad (24)$$

3.2. Long-run plant capacity concepts

CKVDW19 define LR O- and I-oriented PCU concepts under the VRS assumption. In this section, we extend the LR O- and I-oriented PCU concepts to the CRS case. Furthermore, we define an LR attainable O-oriented PCU concept that is new in the literature.

Definition 3.6: LR VRS O-oriented PCU (PCU_o^{LR}) is defined as

$$PCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, VRS) = \frac{DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, VRS)}{DF_o(\mathbf{y}_p \mid P_{\Lambda,VRS})}, \quad (25)$$

where $DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, VRS)$ and $DF_o(\mathbf{y}_p \mid P_{\Lambda,VRS})$ are output efficiency measures relative to technologies including all inputs respectively excluding all inputs.

Proposition 3.3: *The LR O-oriented PCU always exists at the firm level under VRS and under both C and NC assumptions.*

Note that based on Proposition 2.5, we have $DF_o(\mathbf{y}_p \mid P_{\Lambda,CRS}) = \infty$. Hence, Definition 3.6 is invalid under the

CRS case. Therefore, we define an LR O-oriented PCU under the CRS assumption using a pair of O-oriented efficiency measures as follows:

Definition 3.7: LR CRS O-oriented PCU (PCU_o^{LR}) is defined as

$$PCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{x}^\epsilon \mid \Lambda, CRS) = \frac{DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, CRS)}{DF_o(\mathbf{y}_p \mid P^{\mathbf{x}^\epsilon}_{\Lambda,CRS})}, \tag{26}$$

where $DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, VRS)$ and $DF_o(\mathbf{y}_p \mid P^{\mathbf{x}^\epsilon}_{\Lambda,CRS})$ are output efficiency measures relative to technologies including all inputs respectively excluding all inputs bigger or equal to x_ϵ.

Proposition 3.4: *The LR O-oriented PCU (26) always exists at the firm level under CRS and under both C and NC assumptions.*

Note that based on Proposition 2.6, we have $DF_o(\mathbf{y}_p \mid P_{VRS,\Gamma}) = DF_o(\mathbf{y}_p \mid P^{\mathbf{x}^\epsilon}_{VRS,\Gamma})$. Hence, a more general definition of the LR O-oriented PCU which is valid under both VRS and CRS cases can be defined as follows:

$$PCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{x}^\epsilon \mid \Lambda, \Gamma) = \frac{DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)}{DF_o(\mathbf{y}_p \mid P^{\mathbf{x}^\epsilon}_{\Lambda,\Gamma})}. \tag{27}$$

In line with the SR attainable O-oriented PCU in Definition 3.2 discussed above, we can now define a new LR attainable O-oriented PCU notion at the firm level as follows:

Definition 3.8: LR attainable O-oriented PCU ($APCU_o^{LR}$) at attainability level $\bar{\lambda} \in \mathbb{R}_+$ is

$$APCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) = \frac{DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)}{ADF_o(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)},$$

with $DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)$ as defined previously in (8) and where the LR attainable O-oriented efficiency measure $ADF_o(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)$ at a certain attainability level $\bar{\lambda} \in \mathbb{R}_+$ is defined by

$$\begin{aligned} &ADF_o(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) \\ &\quad = \max\{\varphi \mid \varphi \geq 0, 0 \leq \theta \leq \bar{\lambda}, \varphi \mathbf{y}_p \in P_{\Lambda,\Gamma}(\theta \mathbf{x}_p)\}. \end{aligned} \tag{28}$$

For $\bar{\lambda} \geq 1$, since $1 \leq DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma) \leq ADF_o(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)$, note that $0 < APCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) \leq 1$. Also, for $\bar{\lambda} < 1$, since $1 \leq ADF_o(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) \leq DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)$, note that $1 \leq APCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)$.

Proposition 3.5: *The LR attainable O-oriented $APCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma)$ exists at the firm level under both the VRS and CRS cases as well as under both C and NC assumptions.*

Remark 3.2: Note that Propositions 1, 2 and 3 of KSVDW19b can be equally applied to the LR attainable O-oriented PCU.

Definition 3.9: LR I-oriented PCU (PCU_i^{LR}) under VRS is defined as

$$PCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, VRS) = \frac{DF_i(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, VRS)}{DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, VRS)}, \tag{29}$$

where $DF_i(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)$ and $DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, \Gamma)$ are both input efficiency measures aimed at reducing all input dimensions relative to the VRS technology, whereby the latter efficiency measure is evaluated at a zero output level.

Proposition 3.6: *The LR I-oriented PCU always exists at the firm level under VRS.*

Note that based on Proposition 2.7, we have $DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, CRS) = 0$. Hence, Definition 3.9 is invalid under the CRS case. We define an LR I-oriented PCU under the CRS assumption using a couple of I-oriented efficiency measures as follows:

Definition 3.10: LR I-oriented PCU (PCU_i^{LR}) under CRS is defined as

$$PCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS) = \frac{DF_i(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, CRS)}{DF_i(\mathbf{x}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)}, \tag{30}$$

where $DF_i(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, CRS)$ and $DF_i(\mathbf{x}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)$ are the input efficiency measures at the current observed output level and at the $\mathbf{y}^\epsilon$ output level, respectively.

Proposition 3.7: *The LR I-oriented PCU in Definition 3.10 always exists at the firm level under CRS.*

Based on Proposition 2.8, we have $DF_i(\mathbf{x}_p, \mathbf{0} \mid \Lambda, VRS) = DF_i(\mathbf{x}_p, \mathbf{y}^\epsilon \mid \Lambda, VRS)$. Hence, a more general definition of the LR I-oriented PCU which is valid under both VRS and CRS cases can be defined as follows:

$$PCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma) = \frac{DF_i(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)}{DF_i(\mathbf{x}_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)}. \tag{31}$$

3.3. Existence of plant capacity concepts at the firm level: conclusion

Wrapping up, the question regarding the existence of solutions for the SR as well as the LR O-, attainable O-, and I-oriented PCU at the firm level can be answered

Table 1. Summary of firm results in this contribution.

Plant capacity notion*	Returns to scale	Definition	Efficiency measures	Existence firm
$PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, VRS)$[1]	VRS	3.1	(8) and (11)	Yes (Remark 3.1)
$PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, CRS)$[2]	CRS	3.1	(8) and (11)	Yes (Remark 3.1)
$APCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, VRS)$[3]	VRS	3.2	(8) and (20)	Yes (Corollary 3.1)
$APCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, CRS)$	CRS	3.2	(8) and (20)	Yes (Corollary 3.1)
$PCU_{vi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, VRS)$[4]	VRS	3.3	(13) and (16)	Yes (Proposition 3.1)
$PCU_{vi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)$	CRS	3.5	(13) and (19)	No (Sensitivity for $\mathbf{y}^\epsilon$)
$PCU_{fi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, VRS)$	VRS	3.4	(14) and (17)	Yes (Proposition 3.1)
$PCU_{fi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)$	CRS	3.5	(14) and (19)	No (Sensitivity for $\mathbf{y}^\epsilon$)
$PCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, VRS)$[5]	VRS	3.6	(8) and (9)	Yes (Proposition 3.3)
$PCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{x}^\epsilon \mid \Lambda, CRS)$	CRS	3.7	(8) and (10)	No (Sensitivity for $\mathbf{x}^\epsilon$)
$APCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, VRS)$	VRS	3.8	(8) and (28)	Yes (Proposition 3.5)
$APCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, CRS)$	CRS	3.8	(8) and (28)	Yes (Proposition 3.5)
$PCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, VRS)$[6]	VRS	3.9	(12) and (15)	Yes (Proposition 3.6)
$PCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)$	CRS	3.10	(12) and (18)	No (Sensitivity for $\mathbf{y}^\epsilon$)

*Existing plant capacity notions are: [1] Magnussen and Mobley (1999); [2] F88, FGK89 and FGV89; [3] KSVDW19b; [4] CKVDW17; [5,6] CKVDW19. The other plant capacity notions are all new.

affirmatively under both the VRS and CRS cases as well as under both C and NC assumptions. We maintain mild and common axioms on the nonparametric technologies to establish these firm level existence results.

However, while SR and LR O-oriented PCU may well exist from a mathematical viewpoint (Remark 3.1, Propositions 3.3 and 3.4), these concepts may not be attainable: the amounts of variable inputs required to reach the maximum capacity outputs may simply be unavailable at the firm level. Similarly, while solutions for the SR and LR attainable O-oriented PCU may exist (Corollary 3.1 and Proposition 3.5), these concepts may again not be implementable depending on whether the choice of an attainability level $\bar{\lambda}$ is compatible with the real amount of available variable inputs or not. There are no such reservations for the I-oriented PCU.

To facilitate the summary of all key results for the reader, we prepare a summary Table 1 with the results at the firm level that is fairly self-explanatory. It is structured as follows. The first column lists the plant capacity notion. The second column specifies the nature of returns to scale. The third column specifies the equation number of the Definition. The fourth column provides the equation numbers of the efficiency measures involved in the plant capacity definition. The fifth column refers to the existence results regarding the solutions at the firm level. The last column provides either the reference to the existing literature or indicates that the results are new. From the 12 PCU notions under CRS and VRS, only six PCU notions are entirely new in this contribution. We can conclude that at the firm level all PCU notions exist, except for some of the CRS cases.

4. Plant capacity concepts at the industry level

Similar to the firm-level PCU and the question of their existence, it is also possible to devise new SR O-, attainable O-, and I-oriented PCU at the industry level and to check for their eventual existence. Exactly the same existence question pertains to the corresponding LR O-, attainable O-, and I-oriented PCU concepts at the industry level.

4.1. Industry output-oriented plant capacity

Following Proposition 2.1, the constraints on the variable inputs for the SR O-oriented efficiency measure as formulated in (11) are redundant and can be removed from the SR technology $T^f_{\Lambda,\Gamma}$ at the firm level. Therefore, the firms can always consume less or more of its variable inputs to reach the maximum outputs capacity level. But, at the industry level we cannot just remove these variable input constraints.

Indeed, it remains an open question whether there exists a solution for all firms when they reach simultaneously their individual SR O-oriented maximum PCU such that they respect the overall observed variable inputs? In other words, is it possible that all firms reach their full PCU simultaneously while consuming at most the overall amount of observed variable inputs? To answer this question, we formulate the following system of equations:

$$\begin{cases} \sum_{k=1}^{K} \delta z_k^p \mathbf{y}_k \geq DF_o^f(\mathbf{x}^f, \mathbf{y} \mid \Lambda, \Gamma)\mathbf{y}_p, & p = 1, \ldots, K, \\ \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^f \leq \mathbf{x}_p^f, & p = 1, \ldots, K, \\ \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^v \leq \bar{\mathbf{x}}_p^v, & p = 1, \ldots, K, \\ \sum_{p=1}^{K} \bar{\mathbf{x}}_p^v \leq \sum_{p=1}^{K} \mathbf{x}_p^v, & \\ \mathbf{z}^p = (z_1^p \ldots, z_K^p) \in \Lambda, \ \delta \in \Gamma, & p = 1, \ldots, K, \\ \bar{\mathbf{x}}_p^v \geq \mathbf{0}, & p = 1, \ldots, K, \end{cases} \tag{32}$$

where $DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ is the SR O-oriented efficiency measure defined in (11). Note that $\bar{\mathbf{x}}_p^v$ is a decision variable and that $\mathbf{x}_p^v$ is the observed variable input for firm p. Note that formulation (32) is general and applies to both CRS and VRS and to both C and NC technologies. Based on (32), all firms want to simultaneously produce at their maximum capacity and make a trade-off among their variable inputs such that the sum of optimal variable inputs be equal or smaller than the aggregate observed variable inputs (i.e. $\sum_{p=1}^K \bar{\mathbf{x}}_p^v \leq \sum_{p=1}^K \mathbf{x}_p^v$). Note that we reason here in terms of aggregate observed variable inputs: it is equally possible to apply the same reasoning to any aggregate amount of variable inputs that one deems available to the industry.

Remark 4.1: (i) If the industry system of Equations (32) is feasible, then the O-oriented PCU exists at the industry level $IPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ with the given current overall level of variable inputs.
(ii) If the industry system of Equations (32) is infeasible, then the O-oriented PCU at the industry level $IPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ does not exist given the current overall amount of variable inputs.[3]

Note that Färe and Karagiannis (2017, Section 3.3) discuss in a single output context an aggregate O-oriented PCU notion as a weighted sum of individual O-oriented PCU concepts over all firms in an industry. They consider the weighted arithmetic average of individual PCU indices with the weights being potential output shares, defined by projecting the observed output onto the frontier (their method can be generalised into the multiple outputs case when output prices are available). However, their result, in contrast to our analysis, assumes that there are no limits on the aggregate variable inputs at the industry level and that no reallocation of variable inputs occurs across constituent firms.

Note that if the industry system of Equations (32) is infeasible, then we face two options: either the aggregate amount of variable inputs should scale up from the current level (i.e. we need to allocate additional variable inputs to the industry to restore feasibility), or all firms cannot reach simultaneously the maximum PCU level with respect to current overall observed amounts of variable inputs (i.e. some firms must settle for less than full capacity utilisation). We treat these two options sequentially.

First, we assume that there is a possibility at the industry level to obtain some additional variable inputs that can be allocated to the firms. The question arises at least how much additional variable inputs are needed such that all firms are simultaneously able to reach their maximum PCU? To answer this question, we formulate the following model:

$$
\begin{aligned}
& U^I = \min_{\theta, \mathbf{z}^p, \bar{\mathbf{x}}_p^v} \quad \theta \\
& \text{s.t.} \quad \sum_{k=1}^K \delta z_k^p \mathbf{y}_k \geq DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)\mathbf{y}_p, && p = 1, \dots, K, \\
& \sum_{k=1}^K \delta z_k^p \mathbf{x}^f{}_k \leq \mathbf{x}_p^f, && p = 1, \dots, K, \\
& \sum_{k=1}^K \delta z_k^p \mathbf{x}_k^v \leq \bar{\mathbf{x}}_p^v, && p = 1, \dots, K, \\
& \sum_{p=1}^K \bar{\mathbf{x}}_p^v \leq \theta \sum_{p=1}^K \mathbf{x}_p^v, \\
& \mathbf{z}^p \in \Lambda, \ \delta \in \Gamma, && p = 1, \dots, K, \\
& \theta \geq 0, \ \bar{\mathbf{x}}_p^v \geq \mathbf{0}, && p = 1, \dots, K.
\end{aligned} \tag{33}
$$

Note that U^I is interpretable as the minimal expansion of the amount of industry variable inputs needed to be able to produce the full plant capacity outputs for all firms simultaneously.

Proposition 4.1: *Model* (33) *is feasible and* $U^I \leq 1$ *if and only if the system of Equations* (32) *is feasible.*

If $U^I \leq 1$, then all firms can reach their maximum capacities with at most the overall observed variable inputs. If $U^I > 1$, then we need to scale up the industry observed variable inputs by at least U^I such that all firms can reach their maximum capacities. If the existing or available industry variable inputs are at least equal to $U^I \sum_{p=1}^K \bar{\mathbf{x}}_p^v$, then the maximum capacity of all firms can be used at the industry level.

However, as already illustrated and discussed in KSVDW19b, the value of U^I can be quite huge. Therefore, scaling the observed industry variable inputs by an amount U^I may not be attainable at the industry level (see also Section 4.2).

Second, it remains an open question whether there exists a solution for all firms when they optimise their capacity simultaneously without additional variable inputs. We introduce the adjusted industry O-oriented efficiency measure as follows:

Definition 4.1: The adjusted SR industry O-oriented efficiency measure ($\widehat{IDF}_o^f$) for observation $(\mathbf{x}_p, \mathbf{y}_p)$ is

$$\widehat{IDF}_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = \varphi_p^*, \tag{34}$$

where φ_p^* is the optimum value of φ_p in the following model:

$$\begin{aligned}
\max_{\varphi_p,\mathbf{z}^p,\bar{\mathbf{x}}_p^v} \quad & \sum_{p=1}^{K} \varphi_p \\
\text{s.t.} \quad & \sum_{k=1}^{K} \delta z_k^p \mathbf{y}_k \geq \varphi_p \mathbf{y}_p, && p = 1,\ldots,K, \\
& \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^f \leq \mathbf{x}_p^f, && p = 1,\ldots,K, \\
& \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^v \leq \bar{\mathbf{x}}_p^v, && p = 1,\ldots,K, \\
& \sum_{p=1}^{K} \bar{\mathbf{x}}_p^v \leq \sum_{p=1}^{K} \mathbf{x}_p^v, \\
& \mathbf{z}^p \in \Lambda,\ \delta \in \Gamma, && p = 1,\ldots,K, \\
& \varphi_p \geq 0,\ \bar{\mathbf{x}}_p^v \geq \mathbf{0}, && p = 1,\ldots,K,
\end{aligned} \tag{35}$$

where Λ and Γ allow for both C and NC technologies and both CRS and VRS technologies, respectively. Industry model (35) is a central resource allocation model including K linear programs corresponding to each firm with a bogus objective function and a common constraint on the overall observed variable inputs in the industry. Specifically, this single program aims to maximise the O-oriented biased PCU of all firms (φ_p) by reallocating the variable inputs such that the overall observed amount of variable inputs is satisfied.

Let φ_p^{**} be the optimum value of φ_p in industry model (35) without its last functional constraint $\sum_{p=1}^{K} \bar{\mathbf{x}}_p^v \leq \sum_{p=1}^{K} \mathbf{x}_p^v$. In this case, we obtain $DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = \varphi_p^{**}$. Consequently, by ignoring this global industry constraint of model (35), both the industry and firm level O-oriented efficiency measures $\widehat{IDF}_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ and $DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ coincide: $\widehat{IDF}_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = \varphi_p^{**}$. It is important to note that – to the best of our knowledge – we are the first to address the concept of an industry level O-oriented PCU.

Using the SR industry O-oriented efficiency measure (Definition 4.1), one can define the adjusted SR industry O-oriented PCU as follows:

Definition 4.2: The adjusted SR industry O-oriented PCU ($\widehat{IPCU}_o^{SR}$) for observation $(\mathbf{x}_p, \mathbf{y}_p)$ is

$$\widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = \frac{DF_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, \Gamma)}{\widehat{IDF}_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)}.$$

Because $\widehat{IDF}_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) \leq DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$, $\widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) \geq PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$. Therefore, the adjusted SR industry O-oriented PCU measure is larger than or equal to the traditional measure of SR O-oriented PCU. By analogy, we can distinguish between the adjusted SR industry biased plant capacity measure $\widehat{IDF}_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ and the adjusted SR industry unbiased plant capacity measure $\widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$, where the ratio of efficiency measures ensures elimination of any existing inefficiency.

Observe that there is no a priori relation between both the biased and unbiased versions of the SR O-oriented PCU measures at the firm and industry levels. Thus we can write $\widehat{IDF}_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda}) \gtreqless DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$ and $\widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda}) \gtreqless PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$.

Note that based on Proposition 4.4, if $U^I \leq 1$, then the system of Equations (32) is feasible and the O-oriented PCU exists at the industry level with the given current overall level of variable inputs. In this condition, we have $\widehat{IDF}_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$. Therefore, if $U^I \leq 1$, then the adjusted SR industry O-oriented PCU is identical to the SR firm level O-oriented PCU.

Remark 4.2: Note that the same structure as developed in this section can be used to define the LR O-oriented PCU at the industry level. Since we have no partitioning of the inputs in this LR case, hence $N_f = 0$ and $N = N_v$. Therefore, removing the constraints corresponding to the fixed inputs from the system of Equations (32) and in models (33) and (35), and furthermore replacing $DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ with $DF_o(\mathbf{y}_p \mid P_{\Lambda,\Gamma}^{\mathbf{x}^\epsilon})$ in the system of Equations (32) and in model (33) leads to the corresponding concepts for the LR industry O-oriented PCU. As a result, Proposition 4.1 as well as Definitions 4.1 and 4.2 can be defined for the LR O-oriented PCU.

4.2. Industry attainable output-oriented plant capacity

There are sometimes additional variable inputs to allocate to the firms. As mentioned above, if the available additional variable inputs are at least as much as $U^I \sum_{p=1}^{K} \bar{\mathbf{x}}_p^v$ where U^I is the optimal value of model (33) and $\bar{\mathbf{x}}_p^v$ represents the observed variable inputs of firm p, then we can allocate the available additional variable inputs to the firms such that all firms reach their full capacity.

However, consider the situation where the available variable inputs are smaller than the minimum level which is needed to reach full capacity in all firms (i.e. the system (32) is infeasible). In this case, KSVDW19b

(p. 1141) defines the industry attainable O-oriented PCU under the VRS assumption solely. A new, slightly generalised definition of the industry attainable O-oriented efficiency measure can now be defined as follows:

Definition 4.3: The SR industry attainable O-oriented efficiency measure ($IADF_o^f$) for observation $(\mathbf{x}_p, \mathbf{y}_p)$ is

$$IADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = \varphi_p^*, \tag{36}$$

where φ_p^* is the optimum value of φ_p in the following model:

$$\begin{aligned}
\max_{\varphi_p, \mathbf{z}^p, \bar{\mathbf{x}}_p^v} \quad & \sum_{p=1}^{K} \varphi_p \\
\text{s.t.} \quad & \sum_{k=1}^{K} \delta z_k^p \mathbf{y}_k \geq \varphi_p \mathbf{y}_p, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^f \leq \mathbf{x}_p^f, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^v \leq \bar{\mathbf{x}}_p^v, && p = 1, \ldots, K, \\
& \sum_{p=1}^{K} \bar{\mathbf{x}}_p^v \leq \bar{\lambda} \sum_{p=1}^{K} \mathbf{x}_p^v, \\
& \mathbf{z}^p \in \Lambda, \ \delta \in \Gamma, && p = 1, \ldots, K, \\
& \varphi_p \geq 0, \ \bar{\mathbf{x}}_p^v \geq \mathbf{0}, && p = 1, \ldots, K,
\end{aligned} \tag{37}$$

where Λ and Γ allow for both C and NC technologies and both CRS and VRS technologies, respectively. The constraint $\sum_{p=1}^{K} \bar{\mathbf{x}}_p^v \leq \bar{\lambda} \sum_{p=1}^{K} \mathbf{x}_p^v$ shows that the sum of the decision variables $\bar{\mathbf{x}}_p^v$ cannot be higher than the attainable amount of total variable inputs at the industry level. This single program aims to maximise the attainable O-oriented biased PCU of all firms (φ_p) by reallocating the variable inputs such that a portion of the overall observed amount of variable inputs compatible with the attainability level ($\bar{\lambda}$) is satisfied.

Using the SR industry attainable O-oriented efficiency measure of Definition 4.3, the SR industry attainable O-oriented PCU is defined as follows:

Definition 4.4: The SR industry attainable O-oriented PCU ($IAPCU_o^{SR}$) at attainability level $\bar{\lambda} \in \mathbb{R}_+$ for observation $(\mathbf{x}_p, \mathbf{x}_p)$ is

$$IAPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda}) = \frac{DF_o(\mathbf{x}_p, \mathbf{y}_p)}{IADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})}. \tag{38}$$

This Definition 4.4 is based on Definition 8 in KSVDW19b. Note that $IAPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda}) \geq PCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p)$ since their denominators are ranked as follows: $IADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda}) \leq DF_o^f(\mathbf{x}_p^f, \mathbf{y}_p)$. Therefore, the SR industry attainable O-oriented PCU measure is always larger than or equal to the SR O-oriented PCU measure. By analogy, one may differentiate between the SR industry attainable unbiased PCU measure $IAPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$ and the SR industry attainable biased PCU measure $IADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$, whereby the ratio of efficiency measures guarantees removing any existing inefficiency in the former.

Note that there is no determinate relation between both the biased and unbiased versions of the SR attainable O-oriented PCU measures at the firm and industry levels. Thus we obtain $IADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda}) \gtreqless ADF_o^f(\mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$ and $IAPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda}) \gtreqless APCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$.

The attainability level $\bar{\lambda}$ in model (37) can be varied in a subinterval of $(0, \infty)$. To discover the feasible area for $\bar{\lambda}$, KSVDW19b define in their Definition 9 the critical point L^I solely for the VRS case. Here, we formulate a new, slightly more general model to determine this critical point L^I as follows:

$$\begin{aligned}
L^I = \quad \min_{\theta, \mathbf{z}^p, \bar{\mathbf{x}}_p^v} \quad & \theta \\
\text{s.t.} \quad & \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^f \leq \mathbf{x}_p^f, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^v \leq \bar{\mathbf{x}}_p^v, && p = 1, \ldots, K, \\
& \sum_{p=1}^{K} \bar{\mathbf{x}}_p^v \leq \theta \sum_{p=1}^{K} \mathbf{x}_p^v, \\
& \mathbf{z}^p \in \Lambda, \ \delta \in \Gamma, && p = 1, \ldots, K, \\
& \theta \geq 0, \ \bar{\mathbf{x}}_p^v \geq \mathbf{0}, && p = 1, \ldots, K.
\end{aligned} \tag{39}$$

Similar to Proposition 4 of KSVDW19b(p. 1142), we now have the following proposition:

Proposition 4.2: *Industry model (37) is feasible if and only if* $\bar{\lambda} \geq L^I$.

Note that Proposition 4 of KSVDW19b(p. 1142) contains some further details as to the existence of solutions with regard to another critical upper bound U^I (see (33)). Thus under mild conditions on $\bar{\lambda}$ the SR attainable O-oriented PCU does exist at the industry level.

Remark 4.3: Note that there is a relation between the industry attainable output-oriented plant capacity $IAPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$ defined here and the industry adjusted output-oriented plant capacity $\widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ defined in Section 4.1.

- If $\bar{\lambda} = 1$, then $IAPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda}) = \widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$. Thus both industry plant capacity concepts coincide.

- Moreover, if $U^I \leq 1$, then based on Proposition 4.1 the O-oriented PCU $IPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ exists at the industry level with the given current overall level of variable inputs. As a result, if $U^I \leq 1$ and $\bar{\lambda} \geq U^I$, then $\widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = IPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma) = IAPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$. Thus under these conditions all three industry output-oriented plant capacity concepts coincide.

Remark 4.4: Note that the same structure as developed in this section can be used to define the LR attainable O-oriented PCU at the industry level. Since we have no partitioning for the inputs in the LR case, hence $N_f = 0$ and $N = N_v$. Therefore, removing the constraints corresponding to the fixed inputs from industry model (37) leads to the corresponding model of the LR industry attainable O-oriented PCU. As a result, Definitions 4.3 and 4.4 and Proposition 4.2 can be developed for the LR attainable O-oriented PCU.

4.3. Industry input-oriented plant capacity

There are constraints on the variable inputs for the SR I-oriented efficiency measure at the firm level as formulated in (19). Therefore, the firms can always consume less or an equal amount of its variable inputs to reach the minimal outputs $\mathbf{y}^\epsilon$ defining the SR I-oriented PCU level. While Proposition 2.2 allows to remove the output constraints at the firm level, these same constraints cannot be removed at the industry level.

However, it remains an open question whether there exists a solution for all firms when they reach simultaneously their individual SR I-oriented PCU such that they respect the overall amount of observed variable inputs? To answer this question, we formulate the following system of equations:

$$\begin{cases} \sum_{k=1}^{K} \delta z_k^p \mathbf{y}_k \geq \mathbf{y}^\epsilon, & p = 1, \ldots, K, \\ \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^f \leq \mathbf{x}_p^f, & p = 1, \ldots, K, \\ \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^v \leq DF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p^\epsilon \mid \Lambda, \Gamma)\bar{\mathbf{x}}_p^v, & p = 1, \ldots, K, \\ \sum_{p=1}^{K} \bar{\mathbf{x}}_p^v \leq \sum_{p=1}^{K} \mathbf{x}_p^v, & \\ \mathbf{z}^p \in \Lambda,\ \delta \in \Gamma,\ \bar{\mathbf{x}}_p^v \geq \mathbf{0} & p = 1, \ldots, K, \end{cases} \tag{40}$$

where $DF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p^\epsilon \mid \Lambda, \Gamma)$ is the SR I-oriented efficiency measure (19). Note that $\bar{\mathbf{x}}_p^v$ is a decision variable and $\mathbf{x}_p^v$ is the observed variable inputs of firm p. Note that formulation (40) is general: it applies to both CRS and VRS and to both C and NC technologies. Based on (40), all firms want to start working at their full SR I-oriented PCU simultaneously and make a trade-off among their variable inputs such that the sum of optimal variable inputs be equal or smaller than the aggregate observed variable inputs (i.e. $\sum_{p=1}^{K} \bar{\mathbf{x}}_p^v \leq \sum_{p=1}^{K} \mathbf{x}_p^v$).

Proposition 4.3: *The industry system of Equations (40) is feasible.*

Based on Proposition 4.3, the industry system of Equations (40) is always feasible. Hence, all firms can reach their full SR I-oriented PCU by consuming the overall observed variable inputs. Since the industry system of Equations (40) is always feasible, the SR industry I-oriented PCU exists at the current level of industry variable inputs.

This result contrasts with the lack of definite results in Section 4.1 on the existence of the traditional SR O-oriented PCU concept at the industry level. It makes the SR I-oriented PCU concept a valuable alternative to the traditional SR O-oriented PCU notion.

The industry I-oriented PCU can now be defined as follows:

Definition 4.5: The SR industry I-oriented PCU ($IPCU_{vi}^{SR}$) for observation $(\mathbf{x}_p, \mathbf{x}_p)$ is

$$IPCU_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma) = \frac{DF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p \mid \Lambda, \Gamma)}{IDF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)}. \tag{41}$$

In Definition 4.5, the SR industry I-oriented efficiency measure $IDF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)$ for observation $(\mathbf{x}_p, \mathbf{y}_p)$ is measured as

$$IDF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^\epsilon \mid \Lambda, \Gamma) = \theta_p^*, \tag{42}$$

where θ_p^* is the optimum value of θ_p in the following model:

$$\begin{aligned} \min_{\theta_p, \mathbf{z}^p} & \sum_{p=1}^{K} \theta_p \\ \text{s.t.} & \sum_{k=1}^{K} \delta z_k^p \mathbf{y}_k \geq \mathbf{y}^\epsilon, \quad p = 1, \ldots, K, \\ & \sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^f \leq \mathbf{x}_p^f, \quad p = 1, \ldots, K, \end{aligned}$$

$$\sum_{k=1}^{K} \delta z_k^p \mathbf{x}_k^v \leq \theta_p \mathbf{x}_p^v, \quad p = 1, \ldots, K,$$
$$\sum_{p=1}^{K} \theta_p \mathbf{x}_p^v \leq \sum_{p=1}^{K} \mathbf{x}_p^v,$$
$$\mathbf{z}^p \in \Lambda,\ \delta \in \Gamma, \quad p = 1, \ldots, K. \tag{43}$$

In model (43) Λ and Γ allow for both C and NC technologies and both CRS and VRS technologies, respectively. The constraint $\sum_{p=1}^{K} \theta_p \mathbf{x}_p^v \leq \sum_{p=1}^{K} \mathbf{x}_p^v$ shows that the sum of the optimal variable inputs $\theta_p \mathbf{x}_p^v$ cannot be higher than the total amount of total variable inputs at the industry level. This single program minimises the I-oriented biased PCU of all firms (θ_p) by reducing the variable inputs such that the overall observed amount of variable inputs is satisfied.

We now turn to clarify the relation between I-oriented firm and industry biased and unbiased PCU concepts:

Proposition 4.4: *We have*

(i) $DF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^\epsilon \mid \Lambda, \Gamma) = IDF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)$;
(ii) $PCU_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, VRS) = IPCU_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, VRS)$.

Note that based on Proposition 4.4, the SR industry I-oriented biased and unbiased PCUs are identical to the SR firm level I-oriented biased and unbiased PCUs, respectively.

Remark 4.5: Note that the same structure as developed in this section can be used to define the LR I-oriented PCU at the industry level. Since we have no partitioning for the inputs in the LR case, hence $N_f = 0$ and $N = N_v$. Therefore, removing the constraints corresponding to the fixed inputs from industry system of Equations (40) and replacing $DF_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)$ with $DF_i(\mathbf{x}_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)$ leads to the corresponding result for the LR industry I-oriented PCU. As a result, the LR industry I-oriented PCU exists at the current level of industry inputs.

Remark 4.6: Note that the same structure as developed in this subsection can be used to define the SR fixed I-oriented efficiency measure $IDF_{fi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)$ and the corresponding PCU notion $IPCU_{fi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)$ at the industry level.

4.4. Existence of plant capacity concepts at the industry level: conclusion

Wrapping up our results as to the existence of solutions for the industry problem, we can state the following. If the system of Equations (32) is feasible, then the SR O-oriented PCU exists at the industry level with the given current overall level of variable inputs (Remark 4.1): we see that existence and attainability are intimately linked at the industry level. For the SR industry attainable O-oriented PCU, we have shown that industry model (37) is feasible if and only if the attainability level $\bar{\lambda}$ respects a critical parameter L^I (Proposition 4.2). Finally, for the SR industry I-oriented PCU, we have shown that industry model (40) is always feasible (Proposition 4.3). These results also transpose to the LR.

To facilitate the summary of all key results at the industry level for the reader, we have prepared a summary Table 2 that is rather self-explanatory. It is similarly structured as Table 1, except that now the fifth column refers to the existence results regarding the solutions at the industry level. We can conclude that at the industry level only the SR and LR input-oriented PCU notions exist under VRS, except for their CRS cases. All other industry PCU notions simply fail to exist.

5. Plant capacity concepts under CRS: sensitivity for the choice of $\mathbf{x}^\epsilon$ and $\mathbf{y}^\epsilon$

To explain our theoretical developments with regard to the SR I-oriented PCU (24), we discuss Figure 1(a). This two dimensional figure is drawn in variable input and output space. It displays a C VRS technology represented by the polyline *abcd* and the horizontal extension to the right of point *d*. It also displays a CRS cone starting at the origin and passing through point *c*. For an output level $y_{\min}$, point $\bar{p}$ is projected on the cone at point $\bar{b}$. By contrast, for an output level y^ϵ point $\bar{\bar{p}}$ is projected on the cone at point $\bar{a}$ under the CRS case. The latter solution is the closest we can get to the origin of the cone.

Figure 1(b) develops the geometric intuition behind the SR and LR PCU under CRS. The isoquant denoting the combinations of fixed and variable inputs yielding a given output level $L_{C,CRS}(y_p)$ is represented by the polyline *abcd* and its vertical and horizontal extensions at *a* and *d*, respectively. We focus on observation *p* to illustrate first the SR I-oriented PCU: for a given fixed input vector, it seems logical to look for a reduction in variable inputs for given fixed inputs towards the translated point p' that is situated outside the isoquant $L_{C,CRS}(y_p)$ because it produces an output vector $y_{\min}$ (it is compatible with the isoquant $L_{C,CRS}(y_{\min})$ that is situated below the isoquant $L_{C,CRS}(y_p)$). It also seems logical to look for a reduction in

Table 2. Summary of industry results in this contribution.

Plant capacity notion	Returns to scale	Definition	Efficiency measures	Existence industry
$IPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$	VRS	4.2	(8) and (34)	No (Depends on industry amount of variable inputs: Corollary 4.1)
	CRS	4.2	(8) and (34)	No (Depends on industry amount of variable inputs: Corollary 4.1)
$IAPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$[1]	VRS	4.4	(8) and (36)	No (Depends on attainability level $\bar{\lambda}$: Proposition 4.2)
	CRS	4.4	(8) and (36)	No (Depends on attainability level $\bar{\lambda}$: Proposition 4.2)
$IPCU_{vi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)$	VRS	4.5	(13) and (42)	Yes (Proposition 4.3)
	CRS	4.5	(13) and (42)	No (Sensitivity for $\mathbf{y}^\epsilon$)
$IPCU_{fi}^{SR}(\mathbf{x}_p^f, \mathbf{x}_p^v, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)$	VRS	Remark 4.6	Remark 4.6	Yes (Remark 4.6)
	CRS	Remark 4.6	Remark 4.6	No (Sensitivity for $\mathbf{y}^\epsilon$)
$IPCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{x}^\epsilon \mid \Lambda, \Gamma)$	VRS	Remark 4.2	Remark 4.2	Yes (Remark 4.2)
	CRS	Remark 4.2	Remark 4.2	No (Sensitivity for $\mathbf{x}^\epsilon$)
$IAPCU_o^{LR}(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda})$	VRS	Remark 4.4	Remark 4.4	No (Depends on attainability level $\bar{\lambda}$: Remark 4.4)
	CRS	Remark 4.4	Remark 4.4	No (Depends on attainability level $\bar{\lambda}$: Remark 4.4)
$IPCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, \Gamma)$	VRS	Remark 4.5	Remark 4.5	Yes (Remark 4.5)
	CRS	Remark 4.5	Remark 4.5	No (Sensitivity for $\mathbf{y}^\epsilon$)

[1] Reference of $IAPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \bar{\lambda})$ under VRS is KSVDW19b. The other plant capacity notions are new.

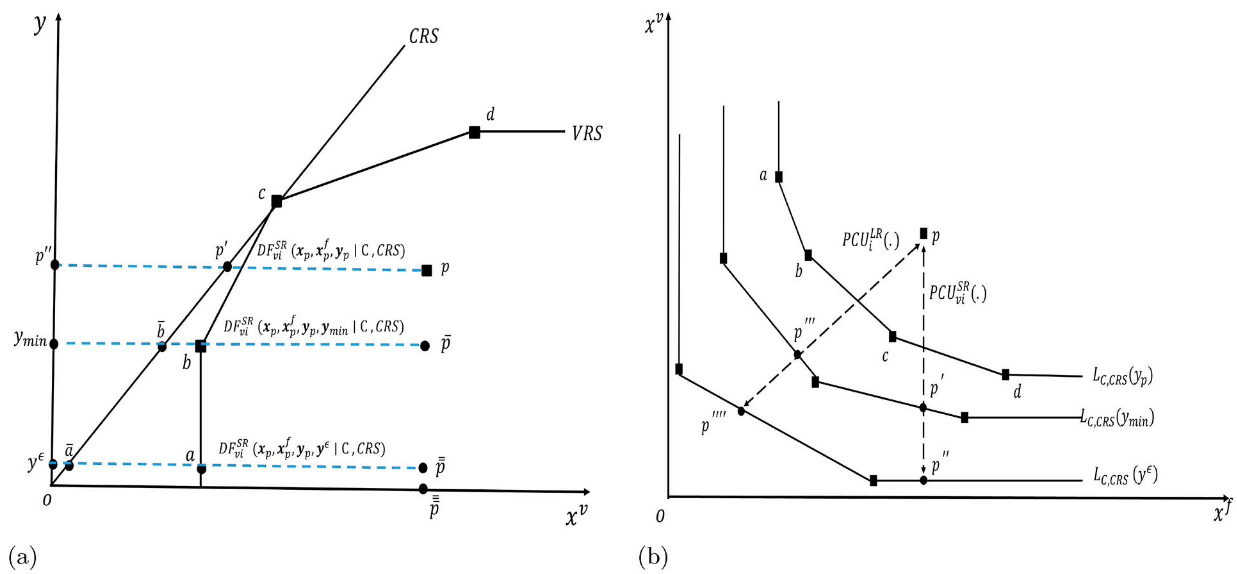

Figure 1. Sensitivity of $PCU_i^{SR}(.)$ and $PCU_i^{LR}(.)$ for the choice of $\mathbf{y}^\epsilon$. (a) Technology with SR I-oriented PCU measures. (b) Isoquant with SR and LR I-oriented PCU measures.

variable inputs for given fixed inputs towards the translated point p'' that is situated outside the isoquant $L(y_p)$ because it produces an output vector y^ϵ (it is compatible with the isoquant $L_{C,CRS}(y^\epsilon)$ that is situated below the isoquants $L_{C,CRS}(y_p)$ and $L_{C,CRS}(y_{\min})$).

Note that we have $PCU_{vi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \mathbf{y}_{\min} \mid \Lambda, CRS) < PCU_{vi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)$ and if $\mathbf{y}^\epsilon$ becomes smaller and smaller, $PCU_{vi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)$ will become bigger and bigger (see Proposition 5.1(i)). Note that the same analysis can be applied to the fixed SR I-oriented $PCU_{fi}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)$.

The LR I-oriented PCU now equally looks for a reduction in all inputs towards the translated point p''' that is situated outside the isoquant $L(y_p)$ because it corresponds to an output level $y_{\min}$. Also, it looks for a reduction in all inputs towards the translated point p'''' that is situated again outside the isoquant $L_{C,CRS}(y_p)$ because it corresponds to an output level y^ϵ.

Note that we have $PCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}_{\min} \mid \Lambda, CRS) < PCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)$ and if $\mathbf{y}^\epsilon$ becomes smaller and smaller, $PCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS)$ will become bigger and bigger (see Proposition 5.1(ii)).

While a solution for the SR and LR I-oriented PCU exists at both the firm and industry levels, there are numerical issues under CRS. Indeed, under CRS one can prove the following result for the SR and LR I-oriented PCU:

Proposition 5.1: *We have*

$$\text{(i)} \quad \lim_{\mathbf{y}^\epsilon \to \mathbf{0}} PCU_{\iota}^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS) = \infty,$$

$$\text{(ii)} \quad \lim_{\mathbf{y}^\epsilon \to \mathbf{0}} PCU_i^{LR}(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}^\epsilon \mid \Lambda, CRS) = \infty. \tag{44}$$

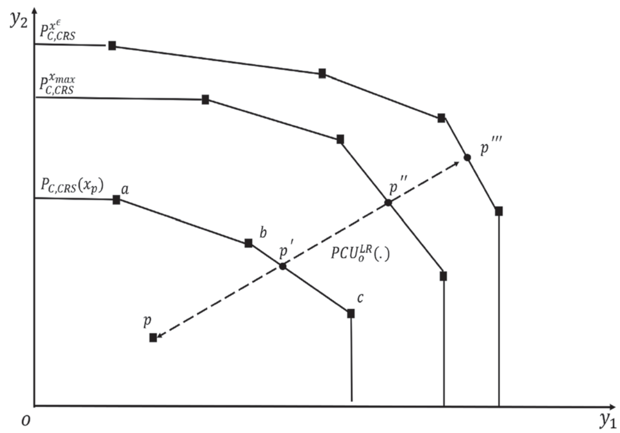

Figure 2. Isoquant with LR O-oriented PCU: Sensitivity of $PCU^{LR}_o(.)$ for the choice of $\mathbf{x}^\epsilon$.

Thus the smaller y^ϵ the more the SR and LR I-oriented PCU become arbitrarily large. This reveals that the above theoretical solution for the CRS case (24) and (31) may face numerical problems.

Obviously, there are rather straightforward solutions to this problem. For instance, if we consider $\mathbf{y}^\epsilon = \mathbf{y}_{\min}$ in the CRS case, then we can define the $PCU^{SR}_i(\mathbf{x}_p, \mathbf{x}^f_p, \mathbf{y}_p, \mathbf{y}_{\min} \mid \Lambda, \Gamma)$ and $PCU^{LR}_i(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}_{\min} \mid \Lambda, CRS)$ under the CRS case, and we can obtain some more reasonable results. Actually, we have $PCU^{SR}_i(\mathbf{x}_p, \mathbf{x}^f_p, \mathbf{y}_p, \mathbf{y}_{\min} \mid \Lambda, CRS) \neq PCU^{SR}_i(\mathbf{x}_p, \mathbf{x}^f_p, \mathbf{y}_p, \mathbf{y}_{\min} \mid \Lambda, VRS)$ as well as $PCU^{LR}_i(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}_{\min} \mid \Lambda, CRS) \neq PCU^{LR}_i(\mathbf{x}_p, \mathbf{y}_p, \mathbf{y}_{\min} \mid \Lambda, VRS)$.

Figure 2 develops the geometric intuition behind the LR O-oriented PCU. The isoquant denoting the combinations of two outputs yielding a given input level $P_{C,CRS}(x_p)$ is represented by the polyline abc and its horizontal and vertical extensions at a and c, respectively. We focus on observation p to illustrate first the LR O-oriented PCU. The LR O-oriented PCU $PCU^{LR}_o(x_p, \mathbf{y}_p, x_{\max} \mid \Lambda, CRS)$ – its corresponding isoquant is labelled $P^{x_{\max}}_{C,CRS}$ in Figure 2 – scales up all inputs at most as much as $x_{\max}$ to reach a translated point p'' that allows maximising the vector of outputs.

In a similar way, the LR O-oriented $PCU^{LR}_o(x_p, \mathbf{y}_p, x^\epsilon \mid \Lambda, CRS)$ – its corresponding isoquant is labelled $P^{x^\epsilon}_{C,CRS}$ in Figure 2 – scales up all inputs at most as much as x^ϵ to reach a translated point p''' that allows maximising the vector of outputs. Note that we have $PCU^{LR}_o(x_p, \mathbf{y}_p, x_{\max} \mid \Lambda, CRS) > PCU^{LR}_o(x_p, \mathbf{y}_p, x^\epsilon \mid \Lambda, CRS)$ and if $\mathbf{x}^\epsilon$ becomes bigger and bigger, $PCU^{LR}_o(x_p, \mathbf{y}_p, x^\epsilon \mid \Lambda, CRS)$ will become smaller and smaller (see Proposition 5.2).

In particular, under CRS one can prove the following result for the LR O-oriented PCU:

Proposition 5.2: *We have:* $\lim_{\mathbf{x}^\epsilon \to \infty} PCU^{LR}_o(\mathbf{x}_p, \mathbf{y}_p, \mathbf{x}^\epsilon \mid \Lambda, CRS) = 0$.

Thus the bigger $\mathbf{x}^\epsilon$ the more the LR O-oriented PCU become arbitrarily small. This reveals that the above theoretical solution for CRS (26) may face numerical problems.

We end with establishing some relations between the LR O-oriented PCU and the LR attainable O-oriented PCU concepts:

Proposition 5.3: *We have*

$$\begin{aligned} &\text{(i)} \quad \lim_{\bar{\lambda}\to\infty} APCU^{LR}_o(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, CRS) \\ &= PCU^{LR}_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, CRS) = 0, \\ &\text{(ii)} \quad \lim_{\bar{\lambda}\to\infty} APCU^{LR}_o(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, VRS) \\ &= PCU^{LR}_o(\mathbf{x}_p, \mathbf{y}_p \mid \Lambda, VRS). \end{aligned} \tag{45}$$

Both LR O-oriented PCU concepts are related to one another when $\bar{\lambda}$ approaches ∞.

Remark 5.1: Note that if we choose $\bar{\lambda}$ and $\mathbf{x}^\epsilon$ such that $\bar{\lambda}\mathbf{x}_p = \mathbf{x}^\epsilon$, then we have $ADF^f_o(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) = DF_o(\mathbf{y}_p \mid P^{\mathbf{x}^\epsilon}_{\Lambda,\Gamma})$. As a result, in this case we have $APCU^{LR}_o(\mathbf{x}_p, \mathbf{y}_p, \bar{\lambda} \mid \Lambda, \Gamma) = PCU^{LR}_o(\mathbf{x}_p, \mathbf{y}_p, \mathbf{x}^\epsilon \mid \Lambda, \Gamma)$.

To conclude this discussion about the computational issues surrounding the above PCU concepts, it is good to consider the following argument. Despite the fact that the seminal contributions of F88, FGK89 and FGV89 define the O-oriented PCU with regard to a CRS technology, one must remember that CRS is unlikely a realistic assumption for any general technology. The CRS assumption implicitly presupposes the economy is in some form of Walrasian general equilibrium. Instead, we consider the more general VRS technology to be the true technology.[4] Therefore, it is preferable to use the VRS assumption to compute any PCU notion. Computational problems related to some of the PCU notions for CRS technologies (see Appendix B for details) are probably minor issues of little relevance for empirical practice. Furthermore, most empirical plant capacity studies impose VRS rather than CRS, as witnessed in the survey discussed in Section 6.

6. Empirical application

We start by reviewing 30 empirical studies in Table 3 employing one or several plant capacity notions.[5] In fact, this selection is based on investigating the citations listed

Table 3. Review of empirical applications.

Article	Plant capacity notions used	Returns to scale	Industry	Country	Remarks
Arfa et al. (2017)	$PCU_o^{SR}(.)$	VRS	Public hospitals	Tunesia	With shadow price constraints
Badau (2015)	$PCU_o^{SR}(.)$	CRS	Industries	USA	Trade resistance model
Cai, Xu, and Yu (2023)	$PCU_o^{LR}(.)$, $PCU_{vi}^{LR}(.)$	VRS	Agriculture	China	By-production model*
Chen and Kerstens (2023)	$PCU_o^{SR}(.)$, $PCU_{vi}^{SR}(.)$	VRS	District courts	Sweden	Horizontal mergers improve plant capacity
Chen, Zhang, and Ni (2020)	$PCU_o^{SR}(.)$	VRS	Provinces	China	Directional distance function & bad outputs
Cui et al. (2023)	$APCU_o^{SR}(.)$	VRS	Universities	China	
Deb (2014)	$PCU_o^{SR}(.)$	VRS	Manufacturing	India	
Dupont et al. (2002)	$PCU_o^{SR}(.)$	VRS	Fisheries	Canada	
Felthoven (2002)	$PCU_o^{SR}(.)$	VRS	Fisheries	USA	Also Stochastic Frontier Analysis
Fukuyama et al. (2021)	$PCU_o^{SR}(.)$	VRS	Steel and iron firms	China	By-production model*
Fukuyama et al. (2022)	$PCU_o^{SR}(.)$	VRS	Steel and iron firms	China	Directional distance function & Kuosmanen (2005) model
Kalai (2019)	$PCU_o^{SR}(.)$	CRS & VRS	Manufacturing	Tunesia	
Karagiannis (2015)	$PCU_o^{SR}(.)$	VRS	Public hospitals	Greece	Second stage regression
Kerr et al. (1999)	$PCU_o^{SR}(.)$	CRS	Acute hospitals	Northern Ireland	
Kerstens and Shen (2021)	$PCU_o^{SR}(.)$, $PCU_{vi}^{SR}(.)$, $PCU_o^{LR}(.)$, $PCU_{vi}^{LR}(.)$	VRS	Hospitals	Hubei province (China)	
Kerstens, Sadeghi, and Van de Woestyne (2019a)	$PCU_o^{SR}(.)$, $PCU_{vi}^{SR}(.)$, $PCU_o^{LR}(.)$, $PCU_{vi}^{LR}(.)$	VRS	Agriculture	France	Also cost-based notions
Kerstens, Squires, and Vestergaard (2005)	$PCU_o^{SR}(.)$	VRS	Fisheries	Denmark	Convex vs. Nonconvex
Kirkley, Paul, and Squires (2002)	$PCU_o^{SR}(.)$	CRS	Fisheries	USA	Also Stochastic Frontier Analysis
Kushwaha, Prawesh, and Venkatesh (2022)	$PCU_o^{SR}(.)$	VRS	Public bus companies	India	Second stage regression
Magnussen and Mobley (1999)	$PCU_o^{SR}(.)$	CRS & VRS	Hospitals	Norway & California	
Shen, Balezentis, and Streimikis (2022)	$PCU_o^{SR}(.)$, $PCU_{vi}^{SR}(.)$, $PCU_o^{LR}(.)$, $PCU_{vi}^{LR}(.)$	VRS	Agriculture	EU states	Directional distance function & Kuosmanen (2005) model
Song et al. (2023)	$PCU_o^{SR}(.)$, $PCU_{vi}^{SR}(.)$, $PCU_o^{LR}(.)$, $PCU_{vi}^{LR}(.)$	VRS	Medical institutions	China	Directional distance function & By-production model*
Song, Ren, and Yang (2023)	$PCU_o^{SR}(.)$	VRS	Universities	China	Directional distance function
Tingley and Pascoe (2005)	$PCU_o^{SR}(.)$	VRS	Fisheries	UK	Second stage regression
Valdmanis, Bernet, and Moises (2010)	$PCU_o^{SR}(.)$	CRS	Hospitals	Florida	Simulation of closure of Miami capacity on state-wide capacity
Valdmanis, Kumanarayake, and Lertiendumrong (2004)	$PCU_o^{SR}(.)$	CRS	Public hospitals	Thailand	
Valdmanis, DeNicola, and Bernet (2015)	$PCU_o^{SR}(.)$	VRS	Public health clinics	Florida	Bootstrap
Walden and Tomberlin (2010)	$PCU_o^{SR}(.)$	VRS	Fisheries	USA	Convex vs. Nonconvex
Yang and Fukuyama (2018)	$PCU_o^{SR}(.)$	VRS	Provinces	China	Directional distance function & bad outputs
Yang, Fukuyama, and Song (2019)	$PCU_o^{SR}(.)$	VRS	Manufacturing	China	Directional distance function & bad outputs
Zhang et al. (2020)	$PCU_o^{SR}(.)$	VRS	Transportation	China	Directional distance function & bad outputs
Zhang et al. (2020)	$PCU_o^{SR}(.)$	VRS	Construction	China	Directional distance function & bad outputs

*Murty, Russell, and Levkoff (2012).

in Google Scholar for the seminal methodological articles of F88, FGK89, FGV89, CKVDW17, KSVDW19b and CKVDW19. We systematically scrutinise these references for eventual empirical applications: we have updated these references till 30 March 2023. This strategy is coherent with the objective to list a substantial series of empirical applications of these various PCU concepts without necessarily being exhaustive.[6]

This table is structured as follows. The first column contains the reference. The second column indicates the plant capacity concept(s) employed: for simplification, the arguments of the function have been suppressed. The third column indicates the postulated returns to scale assumption. The fourth and fifth columns indicate the industry and country of application. A final column contains some additional remarks to indicate any eventual particular features.

Table 3 supports the following conclusions. First, about three quarters of all studies impose VRS rather than CRS (see the third column), supporting our position that CRS is little relevant. Second, while about half of the studies concern hospitals or fisheries, there is now a rather wide series of sectors covered (see the fourth column), showing the general applicability of these PCU concepts. In particular, the economic sectors covered involve primary activities (e.g. agriculture, fisheries, etc.), secondary activities (e.g. construction, manufacturing, steel and iron firms, among others), and tertiary activities (e.g. hospitals, public bus companies, universities, etc.). Finally, while the large majority of studies applies the traditional SR O-oriented PCU notion (denoted $PCU_o^{SR}(.)$), the recent applications focus on two things: (i) the use of alternative PCU notions (see the second column) and (ii) the extension of PCU concepts to the joint production of good and bad outputs (see the final column). Thus overall these features demonstrate the universal viability of the PCU approaches in different settings and the recent spread of alternative PCU concepts.

To illustrate the PCU frameworks developed we use the secondary data set of Fan, Li, and Weersink (1996). These data contain 471 specialised dairy farms from the province of Quebec in Canada. The single output is milk production per cow. The four inputs are: (i) forage consumption; (ii) grain and concentrate consumption; (iii) value of capital stock; and (iv) labour-person units. These inputs are also expressed in units per cow. For the purpose of the analysis, the fixed inputs are capital and labour, and the variable inputs are forage consumption and grain and concentrate consumption. Since the SR attainable O-oriented PCU notion has been empirically illustrated in KSVDW19b and given the difficulty to assign an attainability level ($\bar{\lambda}$), we here focus on the SR industry O-oriented PCU, the adjusted SR industry O-oriented PCU, and the SR industry I-oriented PCU notions.

Table 4 reports descriptive statistics on the SR and LR, input- and output-oriented PCUs at both firm and industry levels under the VRS case. We report the average, the standard deviation, and the minima and maxima depending on the context. The first horizontal part of Table 4 reports the output-oriented results, and the second horizontal part provides the input-oriented results. The second and the third columns are for the SR case, and the two last columns present the results for the LR case. The three last rows focus on comparing firm level and industry-level results. In particular, the first horizontal part shows the number of observed units that have the amounts $PCU_o(.) > \widehat{IPCU}_o(.)$, $PCU_o(.) < \widehat{IPCU}_o(.)$, and $PCU_o(.) = \widehat{IPCU}_o(.)$, respectively. In the second horizontal part the same amounts are shown for the comparison between $PCU_{vi}(.)$ and $IPCU_{vi}(.)$.

By solving model (33), for the SR we have $U^I = 1.42$ ($U^I = 2.61$ for the LR case), therefore based on Proposition 4.1 the industry system of Equations (32) in both the SR and LR cases are infeasible. Thus the industry O-oriented PCU $IPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ does not exist for our empirical sample. Therefore, in the remainder we only focus on the adjusted industry O-oriented PCU $\widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$.

Note that U^I is interpretable as the minimal expansion of the amount of industry variable inputs needed to be able to produce the full plant capacity outputs for all firms simultaneously. As a result, we need to scale up the industry observed variable inputs by at least 1.42 times (2.61 times in the LR case) such that all firms can reach their maximum capacities. However, scaling the observed industry variable inputs by this amount may not be attainable at the industry level (see Section 4.2). To provide a solution for all firms when they optimise their capacity simultaneously without additional variable inputs, we apply the adjusted industry O-oriented efficiency measure $\widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, VRS)$ from Definition 4.2. The descriptive statistics for the results of $\widehat{IPCU}_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, VRS)$ are reported in the third column (for the SR case) and the fifth column (for the LR case) of the first part of Table 4.

Analysing the results in Table 4, we can draw the following conclusions. First, the PCU measure at the firm level indicates that current outputs make up 92% from maximal plant capacity outputs in the SR and 85% in the LR, on average. However, the PCU measure at the industry level shows that current outputs make up higher

Table 4. Descriptive statistics of plant capacity utilisation at firm and industry level.

	SR		LR	
Output-oriented	$PCU_o^{SR}(.)$	$\widehat{IPCU}_o^{SR}(.)$	$PCU_o^{LR}(.)$	$\widehat{IPCU}_o^{LR}(.)$
Average	0.928	0.969	0.853	0.942
Stand. Dev.	0.062	0.076	0.079	0.122
Minimum	0.518	0.518	0.435	0.455
Maximum	1	1.166	1	1.293
$PCU_o(.) > \widehat{IPCU}_o(.)$	0		0	
$PCU_o(.) < \widehat{IPCU}_o(.)$	459		471	
$PCU_o(.) = \widehat{IPCU}_o(.)$	12		0	
	SR		LR	
Input-oriented	$PCU_{vi}^{SR}(.)$	$IPCU_{vi}^{SR}(.)$	$PCU_i^{LR}(.)$	$IPCU_i^{LR}(.)$
Average	1.348	1.348	1.127	1.127
Stand. Dev.	0.276	0.276	0.166	0.166
Minimum	1	1	1	1
Maximum	3.644	3.644	2.873	2.873
$PCU_{vi}(.) > IPCU_{vi}(.)$	0		0	
$PCU_{vi}(.) < IPCU_{vi}(.)$	0		0	
$PCU_{vi}(.) = IPCU_{vi}(.)$	471		471	

than that at the firm level: i.e. 96% in the SR and 94% in the LR, on average. Second, since the industry system of Equations (32) in both SR and LR cases are infeasible, all firms cannot reach simultaneously the maximum PCU level with respect to current overall observed amounts of variable inputs (i.e. some firms must settle for less than full plant capacity utilisation). In this situation for some firms $\widehat{IPCU}_o^{SR}(.) > 1$. As can be seen in Table 4, the adjusted PCU measure at the industry level is higher than one: for the SR case it is equal to 1.166 and for the LR case it is equal to 1.293, while the maximum of PCUs at the firm level in both SR and LR equals unity. Third, whereas in general $\widehat{IPCU}_o^{SR}(.) \gtreqless PCU_o^{SR}(.)$, for the majority of observations we find that $PCU_o^{SR}(.) < \widehat{IPCU}_o^{SR}(.)$ and for 12 observations we find that $PCU_o^{SR}(.) = \widehat{IPCU}_o^{SR}(.)$, while in the LR case for all firms $PCU_o^{LR}(.) < \widehat{IPCU}_o^{LR}(.)$.

Three conclusions emerge with regard to the results of the input-oriented PCUs in the second part of Table 4. First, note that based on Proposition 4.4, the SR industry I-oriented PCU equals the SR firm level I-oriented PCU. Therefore, the second and third columns are identical and the fourth and fifth columns are also identical. Second, there is a great amount of heterogeneity in PCUs at both firm and industry levels, as indicated by the standard deviation. Finally, we have rather plausible results for the input-oriented plant capacity measures. For the SR, an average of 1.34 more variable inputs with current outputs than with zero outputs are required, whereas for the LR 1.127 more inputs with current outputs than with zero outputs are required.

7. Discussion and conclusion

This contribution has first defined the SR and LR versions of the traditional O-oriented, attainable O-oriented, and I-oriented PCU notions. We have first established that all these PCU notions are well defined at the firm level for general nonparametric technologies under both CRS and VRS assumptions and under both C and NC. This has led to some theoretical refinements in the definitions of the I-oriented PCU notions as well as the LR O-oriented PCU concept with regard to a CRS technology. It has also led to the definition of a new LR attainable O-oriented PCU concept.

In addition, we have answered the question as to the existence of the same three PCU concepts at the industry level. First, we establish that the SR O-oriented PCU notion is likely not to exist at the industry level. This result is obviously connected to the attainability issue that triggered the introduction of the SR attainable O-oriented PCU concept in the first place. Therefore, we have introduced an SR adjusted O-oriented industry PCU that is always well defined. Second, the SR attainable O-oriented PCU exists for a proper choice of an attainability level $\bar{\lambda}$. Furthermore, this concept is somewhat related to the SR adjusted O-oriented industry PCU. Third, the SR I-oriented PCU notion always exists at the industry level. Furthermore, these same industry results immediately transpose to the corresponding LR PCU notions.

The empirical applications on Canadian dairy farms have shown the following key results. First, the SR O-oriented PCU notion does not exist for our sample. Second, the adjusted SR O-oriented PCU notion can always be applied. Third, the SR industry I-oriented PCU equals the firm level SR I-oriented PCU.

We also pay attention to the computational issues regarding the definition of the I-oriented PCU notion with respect to a CRS technology. These practical issues are less important than they appear if one realises that the true technology is a VRS technology.

To spell out the implications of our findings, we distinguish between managers and researchers. Managers responsible for a single firm or a single plant within a firm may choose a method that is useful to the firm and its goals: they may only care about existence at the firm level. Managers responsible for several plants need also care about the existence at the industry level since they recognise the interdependencies between the plants. A researcher should ideally care about the existence issues at both the firm and industry levels. Combined with the attainability issue, this leaves in our opinion little choice. In conclusion, in contrast to F88, FGK89, and FGV89, and in line with KSVDW19b, this contribution casts some doubt on the widespread use of the traditional SR O-oriented PCU. If one wants a PCU notion that always exists at both the firm and industry levels, then the SR and LR I-oriented PCU notions are the only choices available.

Our research has been limited to plant capacity concepts. One limitation is that we have completely ignored other capacity concepts based on, for instance, the cost function: see, e.g. Kerstens, Sadeghi, and Van de Woestyne (2019a). Just to sketch one avenue for future work, following Färe and Karagiannis (2017) it could be interesting to investigate the conditions under which aggregate PCU concepts can be defined.

Notes

1. Throughout this contribution, $\mathbb{R}^d$ denotes the d-dimensional Euclidean space, and $\mathbb{R}^d_+$ denotes its non-negative orthant; lowercase boldface letters are used to denote vectors; all vectors are considered to be column vectors and vectors $\mathbf{0}$ denotes vector of zeroes; and for vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^d$, the inequality $\mathbf{a} \geq \mathbf{b}$ ($\mathbf{a} > \mathbf{b}$) means that $a_i \geq b_i$ ($a_i > b_i$), for all $i = 1, \ldots, d$.
2. Note that the convex VRS technology does not satisfy inaction.
3. The industry O-oriented PCU $IPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$ is not formally defined because from a mathematical viewpoint it is not always well-defined. Therefore, we refrain from providing a formal definition. We only use the concept industry O-oriented PCU in an informal way, denoted by the symbol $IPCU_o^{SR}(\mathbf{x}_p, \mathbf{x}_p^f, \mathbf{y}_p \mid \Lambda, \Gamma)$.
4. Scarf (1994, 114–115) mocks the possibility of a CRS technology: 'Both linear programming and the Walrasian model of equilibrium make the fundamental assumption that the production possibility set displays constant or decreasing returns to scale; that there are no economies associated with production at a high scale. I find this an absurd assumption, contradicted by the most casual of observations. Taken literally, the assumption of constant returns to scale in production implies that if technical knowledge were universally available we could all trade only in factors of production, and assemble in our own backyards all of the manufactured goods whose services we would like to consume.'
5. The following empirical comparative studies of our own are not included in Table 3 because these are methodological in nature. CKVDW17 offer a numerical example of SR VRS I-oriented PCU notion and discuss an empirical illustration of SR VRS O- and I-oriented PCU concepts under C and NC. KSVDW19b report an empirical analysis of the SR VRS O- and attainable O-oriented PCU concepts under C and NC. CKVDW19 provide a detailed numerical example as well as an empirical illustration of SR and LR VRS O- and I-oriented PCU notions under C.
6. This boils down to a semi-systematic approach to a literature review (see Snyder 2019), which is sufficient for our purpose.

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No potential conflict of interest was reported by the author(s).

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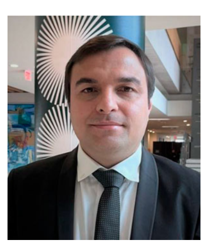

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Data Availability Statement

The data of Fan, Li, and Weersink (1996) have been downloaded at: DOI: 10.1080/07350015.1996.10524675. This data set is made available as supplementary material.

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