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A New Criterion for Technical Efficiency Measures: Non-Monotonicity Across Dimensions Axioms

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Based on non-parametric deterministic production technologies radial and non-radial measures of technical efficiency are evaluated using properties guaranteeing insensitivity to the dimensionality of technology. These new axioms are important in empirical research and may especially prevent manipulation of results when implementing these benchmark methodologies in private or public organizations. An empirical example illustrates to which extent a series of radial and non-radial technical efficiency measures satisfies the proposed axioms. Copyright © 1999 John Wiley & Sons, Ltd.

INTRODUCTION

For non-parametric, deterministic reference technologies it has been shown that radial input (output) measures of technical efficiency cannot decrease when computed on additional input (output) dimensions, and cannot increase if computed on less input (output) dimensions (Nunamaker, 1985; Thrall, 1989). This predictable property of radial efficiency measures creates room for manipulating the results of any performance evaluation. While this problem may seem to some extent remote in academic work, small inaccuracies in linear programming software, for instance, may well induce wrong inferences when comparing model specifications with different dimensionality. Such predictability is clearly of crucial importance when implementing these benchmarking methodologies in both private and public sectors. One can expect a strong pressure to add as many dimensions as possible on the one hand, and to oppose any reduction in the dimensions analysed on the other. It is, therefore, desirable that a measure of technical efficiency is not sensitive to the dimensionality of the production technology. By contrast, there is no such problem in a parametric approach. For instance, in the stochastic composed error framework the radial technical efficiency measure is defined in terms of a residual which by assumption is uncorrelated to all independent variables (see Lovell, 1993).

If the insensitivity of technical efficiency measures to the dimensionality of production technology is indeed a desirable property, an important question is whether alternative efficiency measures defined in the literature do any better in this respect. The major aim of this paper is to verify for a series of efficiency measures whether, and under which conditions, they are insensitive to certain alterations in the dimensionality of technology. The exact insensitivity envisioned and the nature of the alterations considered are more rigorously defined in the main part of the contribution.

In the remainder of this introduction we motivate how the dimensionality of a production technology can bring about problems in applied production analysis, and in efficiency gauging in particular. One can think about three economic reasons why the dimensionality of technology

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may create important judgmental problems. We expand on each of these in turn.

First, the notion of a production technology involves decisions on inputs and outputs and ambiguities may exist on what the proper input and output dimensions are. This includes conceptual problems on how to model a certain production process. For instance, the definition and especially the measurement of banking inputs and outputs is a subject of debate in the literature (see Berger and Humphrey, 1992, or Colwell and Davis, 1992, for thorough discussions). Most popular are the 'production' and the 'intermediation' approaches. The production approach regards banks as producers of deposit and loan accounts using traditional inputs only. Outputs are measured by the numbers of deposit and loan accounts of various types, or by the numbers of transactions on each of these accounts. Under the intermediation approach, by contrast, collected deposits and purchased funds are intermediate inputs for the various types of loans and other assets. Inputs now include purchased funds in addition to the traditional inputs, while outputs are specified as monetary volumes. Thus, both approaches differ especially in the way they measure banking variables.

Furthermore, given the large number of different products and services produced in most modern organizations, it is often inevitable, albeit for data availability reasons alone, to aggregate some input and/or output categories. The aggregation problem is not only a difficult theoretical problem. Of equal importance is that the choice of a proper aggregation level remains very hard to answer in empirical applications (Chambers, 1988). Health care efficiency studies have, for instance, been criticized for their inability to account for the vast heterogeneity of treatments, due to a too high aggregation level. This also causes their results to be of little relevance for policy purposes (Newhouse, 1994).

Conceptual problems are not limited to the specification of technology as such, but can also relate to the goals of production analysis. Especially in evaluating public sector activities, one often distinguishes between efficiency and effectiveness.[1] For example, in analysing hospital services one ideally would like to evaluate the effectiveness of certain interventions on the patient's health status. But lacking proper information on this final output, one is often obliged to limit oneself to study efficiency. Efficiency studies focus on direct outputs, like the number of treated cases per diagnostic group (representing medical services), number of patient days (reflecting 'hotel' services), and number of beds (as a proxy for the option demand) (see Cowing *et al.*, 1983, or Breyer, 1987).

A second economic reason is the distinction between variable and fixed, and between endogenous and exogenous inputs and outputs related to an analysis of production and costs in the short and in the long run (Chambers, 1988). Probably uncontroversial examples of fixed inputs are offices in banking, or hospital buildings and specialized medical equipment (surgery rooms, etc.) in health care. But in many applications it may be difficult to agree on the dimensions that are under managerial control in the short run. The integration of quality dimensions into production analysis provides a typical area for debate. For instance, in health care one may assume that the quality of the outputs is related to the quality of the inputs utilized. Given limited and regulated budgets, a hospital is able to adjust its number of MD's, but it has probably more difficulties adjusting their quality. If this view is agreed upon, then the quantity of MD's should be a variable dimension in the efficiency analysis, while their quality remains a fixed dimension. Furthermore, linking this discussion to the first economic reason, the aggregation problem may induce researchers to culminate quantity and quality dimensions, sometimes putting the usefulness of the analysis at risk. Thus, it is often unclear which dimensions are under managerial control in either the short or the long run.

Finally, the choice between input, output and graph-oriented measures of technical efficiency may cause some controversy, as the goals of organizations are not always evident. This choice between different orientations of measurement is linked to the goals of cost-minimization, revenue maximization and profit maximization, respectively (Färe *et al.*, 1994).[2] Other goals for organizations have been formulated. These are often compatible with one of the three traditional goals. However, the choice of measurement orientation affects not only drastically the efficiency results, but it can also change estimates of production-related characteristics (scale economies, etc.). For example, the vast majority of efficiency studies in banking postulates cost-minimization. More recent papers focus on profit maximization and sometimes yield different conclusions regarding, for instance, scale economies and the role of risk attitudes and risk-related regulatory constraints (Berger *et al.*, 1997; Färe *et al.*, 1997).

After this extensive legitimization of the importance of the dimensionality issue in efficiency analysis, we close this introduction by specifying the structure of our contribution. First, the non-parametric deterministic production technologies and the radial and alternative, non-radial efficiency measures are defined and illustrated in the second and third sections, respectively. This background material clears the ground in the fourth section to provide a clear formulation of the issue from an economic point of view, and to specify exactly new properties guaranteeing a minimal insensitivity to the number of dimensions in the analysis. Two propositions establish to which extent different efficiency measures are insensitive to these particular modifications. In the fifth section, an empirical example, based on a sample of French urban transit operators, illustrates these propositions. A final section concludes.

PRODUCTION TECHNOLOGIES

Assume there are m inputs $(x = (x_1, x_2, \ldots, x_m) \in \Re^m_+)$ producing n outputs $(y = (y_1, y_2, \ldots, y_n) \in \Re^n_+)$ for k observations. Production technology is represented by an input correspondence L: $\Re^n_+ \to 2^{\Re^m_+}$ that maps outputs $y \in \Re^n_+$ into subsets $L(y) \subseteq \Re^m_+$ of inputs. The input set collects all input vectors $x \in \Re^m_+$ that at least produce output vector $y \in \Re^n_+$. This input correspondence is assumed to satisfy the properties discussed in Färe *et al.* (1994).

Alternatively, production technology is defined by its output correspondence P: $\Re^m_+ \to 2^{\Re^n_+}$ mapping inputs $x \in \Re^m_+$ into subsets $P(x) \subseteq \Re^n_+$ of outputs. Finally, the graph of technology is the set of all feasible vectors of inputs and outputs (i.e., the transformation set): $GR = \{(x, y) | x \in L(y), y \in \Re^n_+\} = \{(x, y) | y \in P(x), x \in \Re^m_+\}$. The graph is derived from either the input or the output correspondence, while the latter correspondences can be derived from the graph. Thus, these three definitions provide equivalent characterizations of technology: $x = L(y) \Leftrightarrow y \in P(x) \Leftrightarrow (x, y) \in GR$.

This definition of technology allows for, among others, non-parametric specifications of the production technology. Imposing some basic regularity assumptions, the latter construct a piecewise, (most often) linear technology from observed inputs and outputs. These non-parametric technologies are considered as inner approximations of the true production technology (Varian, 1984; Banker and Maindiratta, 1988). Consequently, measuring technical efficiency with respect to such an inner bound reference technology provides an upper bound on the technical efficiency that could be measured on any other production technology compatible with the data. Examples of this non-parametric approach are the deterministic technologies, based on linear programming approaches (known as Data Envelopment Analysis models in especially the OR literature).

To provide the reader with an idea, we briefly mention some of the more popular technologies within this programming approach (see Färe *et al.*, 1994 for details). For instance, the input correspondence of the variable returns to scale model with strong disposability in both inputs and outputs is defined by:

$$L(y)^{sd-vrs} = \{x | Y^t z \geq y, X^t z \leq x, I^t_k z = 1, z \geq 0\},$$

where Y and X are $k \times n$ and $k \times m$ matrices of observed outputs respectively inputs, z is a $k \times 1$ vector of intensity or activity variables, y and x are $n \times 1$ and $m \times 1$ vectors of outputs respectively inputs, and I_k is a $k \times 1$ unity vector. Other returns to scale assumptions result from appropriate restrictions on z. Piecewise loglinear and CET-CES generalizations are also available. The Free Disposal Hull (FDH)—which imposes no convexity—is obtained from the previous model by restricting z to contain either zeros or ones: $z \in \{0, 1\}$ (see Tulkens, 1993). Other partial relaxations of the convexity assumption have been defined in, among others, Bogetoft (1996). Homothetic piecewise technologies are discussed in Primont and Primont (1994).

Most often these non-parametric specifications of the production technologies are deterministic in nature, although recently stochastic versions have been proposed (for contrasting approaches see Varian, 1990; Ley, 1992; Simar, 1992; Land *et al.*, 1993; Simar and Wilson, 1998). In the remainder of the article we allow for any non-parametric technology based upon mathematical programming approaches.

TECHNICAL EFFICIENCY MEASURES: DEFINITIONS

We consider four technical efficiency measures in the inputs, earlier examined in Färe *et al.* (1983) and in Zieschang (1984). A technical efficiency measure in the inputs $E_i(x, y)$ is defined as a function mapping from the input and output space onto the real line comparing an observed input vector with a smaller feasible input vector for a given output vector. It varies between zero and one, with unity representing efficient production.

Two important subsets denoting production units on the boundary of the technology $L(y)$ are its isoquant ($Isoq\,L(y) = \{x | x \in L(y),\ \lambda x \notin L(y)$ for $\lambda \in [0, 1)\}$), and its efficient subset ($Eff\,L(y) = \{x | x \in L(y), x' \leq x \Rightarrow x' \notin L(y)\}$. Obviously, $Isoq\,L(y) \supseteq Eff\,L(y)$.

The Debreu (1951)–Farrell (1957) radial input measure of technical efficiency is defined:

$$DF_i(x, y) = \min\{\lambda | \lambda \geq 0,\ \lambda x \in L(y)\}.$$

Guaranteeing that production of the same outputs remains feasible, it is the ratio of the smallest feasible contraction of the inputs to the observed input vector itself.

The Färe and Lovell (1978) input technical efficiency measure is defined:

$$FL_i(x, y) = \min\left\{ \sum_{i=1}^{m} \lambda_i / m \,|\, \lambda_i \in (0, 1],\ (\lambda_1 x_1, \ldots, \lambda_m x_m) \in L(y) \right\}.$$

It minimizes the arithmetic mean of scalar reductions in each input dimension. Since each input can be scaled in a different proportion, it is non-radial rather than radial.

The Zieschang (1984) input measure of technical efficiency is specified as:

$$Z_i(x, y) = FL_i(x \cdot DF_i^+(x, y), y) \cdot DF_i^+(x, y)$$

where

$$DF_i^+(x, y) = \min\{\lambda | \lambda \geq 0,\ \lambda x \in L^+(y) = L(y) + \Re_+^m\}.$$

An inefficient observation is first rescaled radially (using $DF_i(x, y)$) to the isoquant of a technology satisfying strong input disposal $L^+(y)$, and then the resulting input vector is shrunk by means of $FL_i(x, y)$ until the efficient subset of $L(y)$ is reached.

Finally, the asymmetric Färe technical efficiency measure is defined as:

$$AF_i(x, y) = \min_{j=1,\ldots,m} \{AF_i^j(x, y)\}$$

where

$$AF_i^j(x, y) = \min\{\lambda_j | \lambda_j \geq 0,\ (x_1, \ldots, \lambda_j x_j, \ldots, x_m) \in L(y)\}.$$

It involves a two-stage minimization process: it takes the minimum over m components $AF_i^j(x, y)$, and each component itself involves a minimization where one input is scaled down holding all other inputs fixed.

To illustrate the practical implications of using these different efficiency measures, we illustrate their impact for an inefficient observation d in Figure 1. This figure is designed such that each efficiency measure projects the inefficient observation to a different point onto the production frontier. Evidently, the latter is no necessity. The radial input efficiency measure computes technical efficiency along a ray through the origin. This clearly leaves an amount of unmeasured efficiency measured by the distance cd' (this shows up as positive slack in the mathematical programming problem). The effect of using non-radial alternatives can be summarized as follows. The Zieschang efficiency measure eliminates any slack starting from the radial projection point d' and leads to point c. The Färe–Lovell measure scales down the inefficient observation to point b. Finally, the asymmetric Färe efficiency measure selects the minimum among the dimensionwise partial measures projecting onto d'' and d'''. In

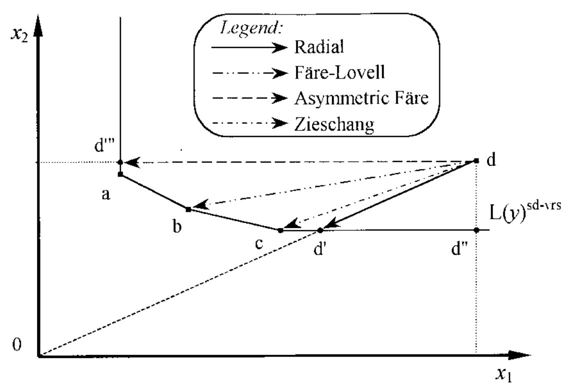

Figure 1. Technical efficiency: the use of different efficiency measures.

this example, the minimum is clearly a projection onto point d''', though this still leaves an amount of unmeasured inefficiency equal to the distance $d'''a$.

The following relation between these four input efficiency measures has been established: $DF_i(x, y) \geq Z_i(x, y) \geq FL_i(x, y) \geq AF_i(x, y)$. Furthermore, when evaluating efficiency over a single input dimension, they all coincide. Other relations between input-oriented, as well as between otherwise oriented, efficiency measures are discussed in De Borger *et al.* (1998).

Färe and Lovell (1978) suggested four properties that an input, and in general, any measure of technical efficiency should satisfy, independent of the technology on which it is applied:

(i) Input vectors are efficient if and only if they belong to the efficient subset.
(ii) Inefficient input vectors are to be compared with respect to vectors in the efficient subset.[3]
(iii) Homogeneity of degree minus one in the inputs.
(iv) Strict negative monotonicity in the inputs.

Other desirable properties, such as unit invariance or commensurability, have been added (see Färe *et al.*, 1994).

While the radial efficiency measure defines technical efficiency relative to the isoquant of $L(y)$ and thus fails to satisfy properties (i) and (ii), its three alternatives obey the Pareto–Koopmans definition of technical efficiency and do focus on the efficient subset of $L(y)$. Only the radial and the Zieschang efficiency measures are homogenous of degree minus one in the inputs. None of the measures satisfies the fourth property in a strict sense, with the Zieschang efficiency measure even being non-monotonous in the inputs.[4]

These four efficiency measures not only serve to measure technical efficiency in a strict sense. They can also be used for decomposing efficiency into various sources. Färe *et al.* (1985, 1994) propose the most elaborate static efficiency taxonomy in the literature and define operational measurement procedures to distinguish between technical, scale, structural (congestion) and allocative efficiency. Both radial and non-radial efficiency measures can be employed to empirically distinguish between these efficiency components.[5] An empirical study disentangling technical, scale and structural efficiency using a non-radial efficiency measure is Viton (1997). In this respect our discussion on the effect of the dimensionality of technology on different efficiency measures is relevant for the whole domain of efficiency and productivity benchmarking.

Our concern for the impact of the dimensionality of technology is related but distinctive from another literature taking a game-theoretic (see Banker *et al.*, 1989) or a principal agent perspective (see Bogetoft, 1994) on the efficiency evaluation problem. These papers recognize that strategic issues arise when organizations are benchmarked on a regular basis by some central authority using any of the existing approaches to efficiency evaluation. Both presuppose a consensus regarding the specification of technology. But Banker *et al.* (1989) explicitly mention, among the possible strategies that may arise in a game context, the following possibilities. (i) Activities may try to become extremely specialized, as this renders them difficult to compare to others. For instance, observations with the minimum value in an input dimension or the maximal value in an output dimension are efficient on a convex production model with variable returns to scale. (ii) Organizations may move to the frontier in a way that leaves them with some organizational slack resources (that remain undetected by the radial efficiency measure). In Figure 1, assuming point d' on the isoquant were an observation, it would appear as efficient when evaluated radially, though there is still an excessive use of inputs equal to the amount cd' in the first input dimension. (iii) Decision-making units may insist on the introduction of new input and/or output dimensions to alleviate the pressure of the efficiency evaluations from the central evaluator. Our contribution as such ignores explicit strategic issues, which may especially occur in a repeated evaluation context. But instead it mainly focuses on the way efficiency is being measured, or on the way different efficiency measures deal with changes in the dimensionality of production technology. In this limited sense it accommodates, among others, the three strategies listed above.

Since the empirical example makes use of output- and graph-oriented efficiency measures, we briefly define radial output and graph efficiency measures. Reasons of space preclude us spelling out in detail analogous definitions for the non-radial efficiency measures. We refer the reader to Färe *et al.* (1985) and De Borger *et al.* (1998).

First, the radial output measure of technical efficiency is defined:

$$DF_o(x, y) = \max\{\lambda | \lambda \geq 1, \lambda y \in P(x)\}.$$

Given a fixed vector of inputs, it measures the ratio of the largest feasible expansion of outputs to the observed output vector itself.

Finally, the radial graph-oriented technical efficiency measure is:

$$DF_g(x, y) = \min\{\lambda | \lambda \geq 0, (\lambda x, \lambda^{-1} y) \in GR\}.$$

It looks simultaneously for a proportional reduction in inputs and a proportional expansion in outputs consistent with the graph of technology.[6]

NON-MONOTONICITY ACROSS DIMENSIONS

The sensitivity of the widely used radial efficiency measure $DF_i(x, y)$ to the dimensionality of the production technology is a problem.[7] Therefore, we add another series of axioms relating to the dimensionality of the production technology.

Introducing a minimum of additional notation, the dimensions of a production technology are elements of a set D. This set D can be partitioned into a subset of respectively variable ($V = (x^V, y^V)$) and fixed ($F = (x^F, y^F)$) dimensions. Note that V may be an improper subset ($V \subseteq D$) but cannot be empty ($V \neq \{\emptyset\}$), while F must be a proper subset ($F \subset D$). The properties are formulated in terms of the relations between two sets representing the dimensionality D respectively D' and the corresponding partitions $\{V, F\}$ and $\{V', F'\}$, where $\# D$ and $\# D'$ indicate the cardinality of the sets D and D', and $E_V(x, y)$ and $E_{V'}(x', y')$ are efficiency measures computed on the variable input and output dimensions defined by these respective partitions. As a regularity condition, we furthermore suppose that there is more than one output dimension and more than one input dimension.

We restate the three economic reasons why this dimensionality issue is important in the above notation. First, the operationalization of the concept of a production technology itself involves decisions on the inputs and the outputs to include. In other words, what are the dimensions to be included in the set D? A second reason is the possible distinction between variable and fixed, or endogenous and exogenous inputs and outputs, i.e. the issue of the variables under control of the organization in the short or the long run. This amounts to a partitioning of the set D. Input and output vectors are, therefore, separated into fixed and variable parts: $x = (x^F, x^V)$, $y = (y^F, y^V)$. Technical efficiency is then evaluated relative to the variable subvectors. A final cause for controversy is the choice between input, output and graph measures of technical efficiency. This amounts to assuming that, respectively, no outputs are variable ($y^V = \{\emptyset\}$), no inputs are variable ($x^V = \{\emptyset\}$), and at least some inputs and outputs are variable ($x^V \neq \{\emptyset\}$ and $y^V \neq \{\emptyset\}$).

Two new properties are proposed requiring that the technical efficiency measure itself be insensitive in specific ways to the total number of dimensions in the production analysis and, in particular, to the number of dimensions over which the efficiency measure is evaluated. Two corresponding propositions summarize which of the efficiency measures reviewed satisfy the property concerned.[8]

First, Non-Monotonicity Across Dimensions Added or Discarded (NOMAD-AD) is a property related to nested technologies where changes in the dimensionality result in two comparable but distinct sets D and D' and in two comparable subsets V and V'.

Definition 4.1: *NOMAD-AD*

For $(x, y) \in \Re_+^{\# D}$ and $(x', y') \in \Re_+^{\# D'}$:

if $D \subset D'$ and

(a) if ($V \subset V'$ and $F \subseteq F'$) or

(b) if ($V = V'$ and $F \subset F'$),

then $E_V(x, y) \lesseqqgtr E_{V'}(x', y')$.

The symmetrical case where $D \supset D'$ can be analogously treated. Observe that $\lesseqqgtr$ denotes a non-monotonic relationship, i.e., it should be logically possible that $E_V(x, y)$ is smaller, equal or larger than $E_{V'}(x', y')$. Otherwise stated, when this relation holds, then some property of solutions is not preserved under some type of changes in dimensionality.

Proposition 4.1:

$FL_V(x, y)$, $Z_V(x, y)$ and $AF_V(x, y)$

satisfy NOMAD-AD Case (a);

None of the four efficiency measures respects NOMAD-AD Case (b).

The first part (a) guarantees for an efficiency measure its insensitivity to simultaneous changes in the total number of input and output dimensions and the total number of variable dimensions over which efficiency measures are evaluated (V and V' are comparable and proper subsets), irrespective of eventual changes in the number of fixed dimensions (F and F' need not be proper subsets). A plausible reason in empirical studies is separability of a production process into several subprocesses (see Chambers, 1988 for a general discussion or Banker, 1992, 1996 for a specific discussion in the context of non-parametric technologies). The second part (b) is of great practical relevance. If the change in the total dimensionality results solely from a change in the fixed dimensions (V is identical to V'; F and F' are comparable and proper subsets), then none of the four efficiency measures remain insensitive to the dimensionality of technology. A typical case is the addition of a series of environmental variables to a given production technology. Practitioners should be aware of its consequences on the resulting efficiency distributions to avoid erroneous conclusions.[9] It raises questions about the validity of any inferences based on a one-stage approach to explaining technical efficiency.[10]

Second, Non-monotonicity Across Dimensions Evaluated (NOMAD-E) is for two identical sets D and D' related to a change in the dimensionality of the partitions $\{V, F\}$ and $\{V', F'\}$.

Definition 4.2: *NOMAD-E*

For $(x, y) \in \Re_+^{\# D}$ and $(x', y') \in \Re_+^{\# D'}$:

if $D = D'$ and

(a) if ($V \subset V'$ and $F \supset F'$) or

(b) if ($V \not\subset V'$ and $F \not\subset F'$),

then $E_V(x, y) \gtreqless E_{V'}(x', y')$.

Proposition 4.2:

$FL_V(x, y)$ and $Z_V(x, y)$

satisfy NOMAD-E Case (a);

$FL_V(x, y)$, $Z_V(x, y)$ and $AF_V(x, y)$

satisfy NOMAD-E Case (b).

The first part (a) guarantees for a production technology of given total dimensionality that an efficiency measure be insensitive to the relative number of variable input and output dimensions. This case includes, for instance, the change from an input or an output to a graph orientation of measurement, or the reverse; and changes in the dimensions evaluated due to the short or long run nature of the analysis (i.e. evaluating efficiency on the whole vector or on a subvector). For a production technology of given dimensionality, the second part (b) requires an efficiency measure to be insensitive to changes in specification resulting in non-comparable subsets of variable and fixed dimensions. A typical example for this second part is a switch from an input to an output orientation of measurement ($V = F'$ and $V' = F$), or the reverse. The effect of switching the orientation of measurement depends on the position of the observation relative to the frontier. This position depends, among others, on the maintained returns to scale assumption. Since radial input and output efficiency measures are identical when evaluated with respect to constant returns technologies (see Färe and Lovell, 1978, or Førsund and Hjalmarsson, 1979), the radial measure in general does not satisfy NOMAD-E but is invariant.

For the case of non-nested or distinct production technologies—characterized by two non-comparable sets D and D'—all efficiency measures are in general non-monotonic with respect to any changes in dimensionality. This trivially holds true since efficiency measurement relative to distinct production technologies involves solving distinct programming problems.

This last case also includes problems of aggregation and disaggregation over variable input or output dimensions. Färe and Lovell (1988) have proven that the radial input efficiency measure is invariant with respect to input aggregation when the cost function is separable in outputs and the corresponding input price indexes. In addition, for this invariance property to hold, the subvectors of input quantity and input price vectors should have been transformed into economic input quantity and input price indexes satisfying certain properties. This invariance result does not apply to any non-radial efficiency measure.

However, if these stringent conditions are not met and aggregation procedures are applied that amount to elementary transformations on the constraints of the associated linear programming problems, whereby the dimensions concerned are variable in both disaggregated and aggregated models, then in general the radial efficiency measure is monotonously affected while all non-radial efficiency measures satisfy non-monotonicity.[11] If, by contrast, the same types of aggregation procedures are applied to fixed dimensions, then all efficiency measures are monotonously affected. Ahn and Seiford (1993) illustrate these trivial results for the radial efficiency measure.

EMPIRICAL EXAMPLE

The empirical illustration uses a sample of French urban transit firms analysed earlier in Kerstens (1996). This sample is selected for illustrative purposes only, because the transport sector offers ample possibilities to document all aspects of the dimensionality issue.

A brief description of the sample follows.[12] The data set contains 114 single mode urban transport companies operating in 1990 outside the Paris region. All operators drive buses only (any other mode has been excluded). The institutional environment is briefly summarized. During a certain period, an urban transport operator supplies transport service within a transport perimeter agreed upon with a public organizing authority (a municipality or group thereof). This authority often owns infrastructure, equipment and rolling stock. The transport perimeter is not limited by territorial boundaries, and only distinguishes urban from interurban transport.

To illustrate how dimensionality issues also affect the specification of transport technologies, we first systematically document how the three economic reasons, mentioned before, play a role in the transportation literature. Then we develop empirical strategies to exemplify our two propositions presented in the previous section.

Technology Specifications in Urban Transport

The very notion of a production technology is a source of some controversy in transportation. Traditional outputs used to model transport technology are: vehicle kilometres; seat kilometres; the number of passengers; and passenger kilometres (see Berechman, 1993). To limit the discussion to the first two and last two outputs: the first two outputs are classical units times distance per unit time concepts and are pure supply indicators; while the last two outputs are demand-related output measures reflecting the effective use of the offered services. This induces some authors (e.g., Chu *et al.*, 1992) to gauge the effectiveness of urban transit instead of its efficiency. Among the traditional inputs are: the number of vehicles; the number of employees; fuel consumption; etc.

In the traditional parametric literature, the above output specifications are often complemented with variables accounting for spatial, temporal and quality characteristics of urban transit services (Jara Díaz, 1982). Following Spady and Friedlaender (1978), this hedonic approach includes additional dimensions (e.g., network length, peak to base ratios, breakdowns, etc.), representing these network characteristics, into the technology specification. However, the non-parametric deterministic reference technologies, relevant to our study, have serious problems to account for these characteristics. In particular, there is no general way to determine sign and significance of any additional dimensions (see Lovell, 1994, or Kerstens and Vanden Eeckaut, 1995b).

In some parametric frontier studies, like the one by Kumbhakar and Bhattacharyya (1996), a variable cost function is being estimated. Other authors (e.g., Fazioli *et al.*, 1993) opt for a total cost frontier. Non-parametric studies, by contrast, assume most of the time that all dimensions of the input or output vector are under managerial control. Few have implemented a subvector efficiency approach.

Most often urban transit studies postulate cost-minimization as a behavioural goal and consequently measure efficiency in the input orientation (e.g., Levaggi, 1994). Cost-minimization is traditionally conceived as a goal for public or private regulated urban transit operators compatible with any other objective pursued by these transport firms (Berechman, 1993). Other papers opt for an output orientation and look for service improvements for given resource constraints (e.g., Kerstens, 1996). Some authors (e.g., Viton, 1997) report both input- and output-oriented efficiency results.

Empirical Strategies

To provide a systematic illustration of the two new properties, we first define a standard specification of technology. Then we develop for each part of the NOMAD-AD and the NOMAD-E definitions a representative case. These scenarios are contrasted against the standard specification to control whether the four efficiency measures indeed can increase, decrease or remain constant. These illustrations are, obviously, to some extent limited by the variables available to us. Except for one scenario (see below), our technology is a non-convex FDH.

The standard specification follows closely the traditional outputs and inputs used to model an urban transit production technology. The output is the number of vehicle kilometres. This pure supply indicator is combined with the following four inputs: average number of vehicles in use; average number of drivers; average number of other employees; and total fuel consumption. This definition of inputs closely follows tradition in the transportation literature. Efficiency is evaluated over all four input dimensions.

The alternative scenarios can be summarized as follows. To illustrate NOMAD-AD part (a), we assume that the production process actually consists of two subtechnologies: one for driving, and another for other activities (administration, maintenance, etc.). Since we have incomplete information on the other activities (some inputs and all outputs are missing), we concentrate on bus driving. Therefore, we eliminate the non-driving personnel dimension from the technology (*SCENARIO 1*). Efficiency is measured over all three remaining input dimensions. The scenario developed for the (b) part of this axiom is to add an environmental variable to the standard specification (*SCENARIO 2*). In particular, following Levaggi (1994) and Viton (1997) among others, we include the total length of the network (all lines) in the production model and treat this variable as an input. In this case, efficiency is measured over all four traditional input dimensions, but not with regard to this environmental characteristic. Since the network is part of the agreement between operator and public organizing authority, it is normally outside the company's control.

NOMAD-E part (a) is exemplified by focusing on subvector efficiency in the standard specification (*SCENARIO 3*). We assume that in the short term only labour and fuel are under managerial control. The stock of vehicles, by contrast, is a fixed dimension. Part (b) is illustrated in two ways (*SCENARIO 4a & 4b*). The first subscenario consists of a mixture of modifying the measurement orientation and the subvector efficiency scenario. We opt for a graph orientation, looking for a simultaneous reduction of inputs and expansion of outputs, but part of the input vector of the standard specification is excluded from the efficiency evaluation. Again vehicles are considered as an exogenous input dimension. The second subscenario considers a switch of measurement orientation, from input to output, on a constant returns to scale version of the FDH technology (see Bogetoft, 1996; Kerstens and Vanden Eeckaut, 1998).[13] But since our standard specification has only one output, it follows that all four output-oriented efficiency measures coincide ($DF_o(x, y) = Z_o(x, y) = FL_o(x, y) = AF_o(x, y)$). Combined with the fact that $1 \geq DF_i(x, y) \geq Z_i(x, y) \geq FL_i(x, y) \geq AF_i(x, y) \geq 0$, this would induce monotonicity for the non-radial efficiency measures. To make the example illustrative we, therefore, add another output dimension: the number of passengers. This is not uncommon in transport models (see, e.g., Viton, 1997).

Empirical Results

Table 1 presents the input efficiency results for the standard specification. Next to descriptive statistics, the number of efficient and inefficient observations is reported for each efficiency measure. Tables 2–6 present the same results for the alternative scenarios, but indicate in addition the number of observations for which the efficiency score increased, remained constant respectively decreased. The latter information is of course crucial to assess the two propositions.

The standard specification in Table 1 yields means between different efficiency measures reflecting the order between them indicated before. The distributions are highly skewed with a long tail to the left. In this sample, $FL_i(x, y)$ and $Z_i(x, y)$ do not differ, though this is no necessity (see, e.g., De Borger *et al.*, 1998, or Ferrier *et al.*, 1994, for empirical evidence). The number of efficient observations for the radial efficiency score is higher than for the three other measures, because

Table 1. Technical Efficiency for the Standard Specification of Technology

	$DF_i(x, y)$	$FL_i(x, y)$	$Z_i(x, y)$	$AF_i(x, y)$
Mean	0.9858	0.9492	0.9492	0.9108
Median	1.0000	1.0000	1.0000	1.0000
Trimmed mean*	0.9903	0.9552	0.9552	0.9204
S.D.	0.0478	0.1108	0.1108	0.1923
Skewness	−4.3921	−2.0673	−2.0673	1.9742
Minimum	0.6667	0.5457	0.5457	0.2400
# Efficient observations	99	91	91	91

* 5% trimmed observations.

it is defined relative to the isoquant, not to the efficient subset.

As can be seen from Table 2, imposing separability and concentrating on driving activities (SCENARIO 1) leads to an downward shift in the mean of all efficiency distributions. The skewness becomes less pronounced. Also the relative number of efficient observations decreases. The example makes clear that, except for the radial efficiency measure that cannot increase, all three non-radial efficiency measures do satisfy NOMAD-AD case (a). This example also indicates that seemingly uniform changes in distributions may hide diverging underlying movements at the individual level.

Table 3 reveals that adding an environmental dimension that remains exogenous to the efficiency evaluation (i.e., SCENARIO 2) shifts all efficiency distributions up and increases the amount of efficient observations. Compared to the standard specification, none of the efficiency measures is able to decrease under these circumstances. Hence, all efficiency scores change in a monotonous way and violate NOMAD-AD case (b).

As evidenced in Table 4, SCENARIO 3 (i.e., a change in the subvector of dimensions under evaluation for a given total number of dimensions) leads to a downward shift in the efficiency distributions, except for the asymmetric Färe indicator. The number of radially efficient observations decreases slightly, but the numbers of efficient DMUs relative to the efficient subset remains constant. Clearly, the radial and the asymmetric Färe efficiency measures are unable to meet the NOMAD-E case (a), though they move in opposite directions. Only the Färe–Lovell and Zieschang efficiency measures are capable of moving in a non-monotonous way. In fact, they again coincide for this particular sample.

Table 5 shows that in SCENARIO 4a all efficiency distributions are slightly shifted upwards. At the individual level, however, all measures meet the requirement of Definition 4.2 (b). Again the number of efficient observations relative to the efficient subset stays the same, while it decreases in the radial case.

SCENARIO 4b is illustrated on a technology with two outputs and four inputs. The four input efficiency measures reported in the first part of

Table 2. Technical Efficiency under SCENARIO 1

	$DF_i(x, y)$	$FL_i(x, y)$	$Z_i(x, y)$	$AF_i(x, y)$
Mean	0.9740	0.9395	0.9395	0.9078
Median	1.0000	1.0000	1.0000	1.0000
Trimmed mean*	0.9784	0.9449	0.9449	0.9151
S.D.	0.0652	0.1118	0.1118	0.1672
Skewness	−2.7735	−1.6957	−1.6957	−1.6283
Minimum	0.6667	0.5382	0.5382	0.4074
# Efficient observations	90	82	82	82
# Obs. with increasing efficiency	0	16	16	12
# Obs. with constant efficiency	104	82	82	92
# Obs. with decreasing efficiency	10	16	16	10

* 5% trimmed observations.

Table 3. Technical Efficiency under SCENARIO 2

	$DF_i(x, y)$	$FL_i(x, y)$	$Z_i(x, y)$	$AF_i(x, y)$
Mean	0.9981	0.9866	0.9866	0.9750
Median	1.0000	1.0000	1.0000	1.0000
Trimmed mean*	0.9995	0.9915	0.9915	0.9846
S.D.	0.0110	0.0535	0.0535	0.1019
Skewness	−6.9211	−4.3367	−4.3367	−4.4402
Minimum	0.9048	0.6864	0.6864	0.4000
# Efficient Observations	110	106	106	106
# Obs. with increasing efficiency	11	17	17	16
# Obs. with constant efficiency	103	97	97	98
# Obs. with decreasing efficiency	0	0	0	0

* 5% trimmed observations.

Table 6 should be compared to the corresponding output measures in the second part of this table. Clearly, the radial efficiency measure is invariant when changing measurement orientation, while all non-radial efficiency measures can vary in a non-monotonous way. In this scenario, a non-convex constant returns to scale technology was employed. But the same would hold true under a more conventional convex constant returns to scale production model.

CONCLUSIONS

This article looks at the issue of the effect of the dimensionality of technology specifications on efficiency measurement. Defining the traditional radial efficiency measure as well as a series of alternative non-radial efficiency measures, we systematically investigated the dimensionality issue on non-parametric (mathematical programming) technologies. We defined two new axioms guaranteeing a non-monotonicity across dimensions added and discarded respectively across dimensions evaluated. The first definition relates to specification changes resulting in nested technologies, while the second applies to technologies of identical dimensionality. The first definition requires for an efficiency measure its insensitivity to: (a) changes in the total number of variable dimensions irrespective of eventual changes in the number of fixed dimensions; (b) changes in the total number of fixed dimensions given no changes in the variable dimensions. The second definition requires for an efficiency measure its insensitivity to: (a) the relative number of variable input and output dimensions; (b) changes in specification resulting in non-comparable subsets of variable and fixed dimensions.

As an overall conclusion, insensitivity for specific changes in the dimensionality of the production technology cannot be guaranteed for all technical efficiency measures. It turns out that the Färe–Lovell and the Zieschang technical efficiency measures perform best in this respect, while the traditional Debreu–Farrell is least satisfac-

Table 4. Technical Efficiency under SCENARIO 3

	$DF_i(x, y)$	$FL_i(x, y)$	$Z_i(x, y)$	$AF_i(x, y)$
Mean	0.9681	0.9423	0.9423	0.9123
Median	1.0000	1.0000	1.0000	1.0000
Trimmed mean*	0.9752	0.9495	0.9495	0.9219
S.D.	0.0947	0.1300	0.1300	0.1901
Skewness	−3.2877	−2.2386	−2.2386	−2.0053
Minimum	0.5198	0.4723	0.4723	0.2400
# Efficient observations	97	91	91	91
# Obs. with increasing efficiency	0	6	6	3
# Obs. with constant efficiency	102	91	91	111
# Obs. with decreasing efficiency	12	17	17	0

* 5% trimmed observations.

Table 5. Technical Efficiency under SCENARIO 4a

	$DF_g(x, y)$	$FL_g(x, y)$	$Z_g(x, y)$	$AF_g(x, y)$
Mean	0.9924	0.9535	0.9535	0.9108
Median	1.0000	1.0000	1.0000	1.0000
Trimmed mean*	0.9947	0.9592	0.9592	0.9204
S.D.	0.0238	0.1031	0.1031	0.1914
Skewness	−3.9702	−2.1784	−2.1784	−1.9610
Minimum	0.8532	0.5893	0.5893	0.2400
# Efficient observations	97	91	91	91
# Obs. with increasing efficiency	10	17	17	2
# Obs. with constant efficiency	99	91	91	111
# Obs. with decreasing efficiency	5	6	6	1

* 5% trimmed observations.

Table 6. Technical Efficiency under SCENARIO 4b

	$DF_i(x, y)$	$FL_i(x, y)$	$Z_i(x, y)$	$Af_i(x, y)$
Mean	0.9352	0.8528	0.8528	0.7279
Median	0.9733	0.8356	0.8356	0.7023
Trimmed mean*	0.9388	0.8575	0.8575	0.7335
S.D.	0.0810	0.1444	0.1444	0.2621
Skewness	−1.2970	−0.5653	−0.5653	−0.3872
	$DF_o(x, y)$	$FL_o(x, y)$	$Z_o(x, y)$	$AF_o(x, y)$
Minimum	0.6194	0.4163	0.4163	0.1420
Mean	0.9352	0.8950	0.8950	0.8433
Median	0.9733	0.9161	0.9161	0.8880
Trimmed mean*	0.9388	0.8992	0.8992	0.8516
S.D.	0.0810	0.1115	0.1115	0.1686
Skewness	−1.2970	−0.8272	−0.8272	−1.0564
Minimum	0.6194	0.5266	0.5266	0.1886
# Efficient observations	46	46	46	46
# Obs. with increasing efficiency	0	11	11	7
# Obs. with constant efficiency	114	46	46	46
# Obs. with decreasing efficiency	0	57	57	61

* 5% trimmed observations.

tory. It is crucial that practitioners are aware of their choice of efficiency measure when benchmarking the performance of organizations. To illuminate this dimensionality issue, the impact of a series of scenarios, relevant for applied production analysis, has been illustrated on a sample of French urban transit companies.

As a closing remark, while it seems to us that insensitivity for changes in the dimensionality of production technology is a desirable property in the efficiency evaluation context, it may well be that in other areas of economics, the same does not hold true. Unaware of any discussion on this topic, we sincerely hope that this article would contribute to a debate as to whether a sensitivity of solutions to the dimensionality of decision-making problems is desirable.

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NOTES

1. These goals traditionally translate into different output specifications: direct outputs (D-output) versus consumer related outputs (C-output) (see Bradford *et al*., 1969). Effectiveness reflects the economic motive for providing services. Ideally, one could evaluate allocative efficiency, but this is very difficult for the public sector or for regulated industries. Furthermore, it is often hampered by a lack of adequate price information. Effectiveness can, therefore, be interpreted as a shortcut to a more extensive evaluation.

2. As indicated in Färe *et al*. (1985), the profit interpretation of graph-oriented efficiency measures is only approximate. Only the recent introduction of directional distance measures (also see endnote 6) has led to proper profit interpretations when using non-parametric technologies.
3. Russell (1985) shows how this property can be made redundant.
4. See Russell (1985), Lovell (1993) and Kerstens and Vanden Eeckaut (1995a) for details. Kerstens and Vanden Eeckaut (1995a) indicate that these non-radial measures include a wide range of alternatives presented in the OR literature.
5. An exception is that the congestion component cannot be straightforwardly evaluated using non-radial efficiency measures (see Dervaux *et al*., 1998).
6. Recently, Chambers *et al*. (1998) provided an alternative basis for efficiency measurement using proper distance concepts instead of ratio measures. Their use of the directional technology distance function leads, among others, to a normalized profit interpretation and contains all known efficiency measures as special cases (the latter is shown in Färe and Grosskopf, 1997). However, since the directional distance measures, like the radial ratio measures, also imply scalar objective functions in the corresponding linear programming problems, everything stated in our paper with regard to the radial efficiency measure also applies to these innovative contributions.
7. The problem is exacerbated in the non-parametric approach by the lack of general test procedures to which a practitioner can turn. Some tests, like the one developed by Brockett and Golany (1996) to evaluate the differences between treatment and control groups, are very specific. Kittelsen (1993) proposes a test procedure based on statistics presented in Banker (1993). The latter assume specific monotone one-sided distributions for the underlying technical inefficiency. Two points are worth stressing. First, only under these specific assumptions can the deterministic non-parametric methodology be interpreted as yielding estimates resulting from a ML estimator. Second, in applications the Kittelsen (1993) procedure seems to have little discriminatory power. Recently, Banker (1996) extended these procedures to test, among others, model specification (e.g., variables to be added at the margin). However, it is fair to say that there is no consensus in the literature and that developments have hardly affected practitioners in the field.
8. Proofs are based on maximal value properties of the programming problems used to compute the technical efficiency measures on the non-parametric production technologies. They are outlined in an appendix that is available upon request.
9. Obeng (1994) is an example of a study making wrong inferences from a comparison between technologies without and with environmental variables (see Kerstens and Vanden Eeckaut, 1995b).
10. See Lovell (1993) for a discussion of one-stage versus two-stage formulations of explanatory models. Yu (1998) reports Monte Carlo simulation results comparing one-stage and two-stage approaches for both non-parametric and parametric frontier methods. Her results suggest that parametric methods deliver more reliable efficiency estimates when environmental variables can be properly specified. Furthermore, one-stage and two-stage approaches deliver extremely poorly for non-parametric frontiers when the magnitude of the effect of the environmental variables on bridging the gap between the kernel production function and the frontier is high.
11. Elementary transformations in linear programming are discussed in Goldman and Tucker (1956).
12. For all details the reader is referred to Kerstens (1996).
13. For the ease of comparison, all output efficiency measures are redefined to be smaller or equal to unity.

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