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METHODS

Plant Capacity and Attainability: Exploration and Remedies

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Abstract. The output-oriented plant capacity notion, which has been around for more than two decades, has been empirically applied mainly in the fishery and the hospital sectors. Since its introduction to the literature, a specified problem is that this notion may not be attainable, in that it presupposes potentially unlimited numbers of variable inputs to determine the maximum number of outputs available. However, this lack of attainability has not been explored previously. This paper fills this void both theoretically and empirically by showing that attainability may be problematic and that bounds on the numbers of variable inputs may well need to be imposed.

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1. Introduction

Research in the economics literature has developed a variety of capacity notions (see, e.g., Johansen 1987 or Nelson 1989). One useful taxonomy distinguishes between technical or engineering notions and economic capacity concepts, the latter of which are mainly based on or derived from some cost function. This paper focuses on the plant capacity notion that is part of the family of technical or engineering notions.

Johansen (1987, p. 362) defined the notion of plant capacity informally as "the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." Färe et al. (1989a, b) translated this plant capacity notion into, respectively, a single- and multiple-output nonparametric frontier framework that helps determine capacity as well as a measure of capacity utilization from information on observed inputs and outputs using a pair of output-oriented efficiency measures.

For more than two decades, research has employed empirical applications using this output-oriented plant capacity mainly in the fishery (e.g., Felthoven 2002, Tingley and Pascoe 2005, Walden and Tomberlin 2010, or Pascoe et al. 2013) and hospital (e.g., Kerr et al. 1999; Valdmanis et al. 2010, 2015; or Karagiannis 2015) sectors. Although one study also focuses on banking (e.g., Sahoo and Tone 2009) and another describes a macroeconomic application on trade barriers (e.g., Badau 2015), to our knowledge, no major methodological innovation has occurred related to this plant capacity concept. However, Cesaroni et al. (2017) recently used the same nonparametric frontier framework to define a new input-oriented measure of plant capacity utilization based on a couple of input-oriented efficiency measures.

Johansen (1987) argued that the plant capacity concept need not necessarily be attainable, in that the numbers of variable inputs required to determine the maximum potential outputs may well be unavailable at both the firm and sector levels. To the best of our knowledge, the literature has completely ignored this issue of attainability. Thus, the main goal of this paper is to explore this attainability problem. At the theoretical level, we argue that there is indeed such an issue for the output-oriented plant capacity notion, but we also show that the new input-oriented plant capacity concept does not suffer from this problem. At the empirical level, we illustrate the extent to which the numbers of variable inputs required to determine the plant capacity output are plausible or not using a secondary data set.

It is well known that the axiom of convexity has a potentially large impact on empirical analyses based on technologies (e.g., Tone and Sahoo 2003). For example, in the context of plant capacity utilization, Walden and Tomberlin (2010) empirically illustrate the effect of convexity on the output-oriented plant capacity notion. Similarly, Cesaroni et al. (2017) compare output- and input-oriented plant capacity concepts empirically and reveal the influence of convexity on both the output- and input-oriented plant capacity concepts. Therefore, we also analyze the issue of attainability in terms of the potential effect of the convexity axiom.

The structure of the rest of this contribution is as follows. Section 2 provides the basic definitions of technology and the efficiency measures representing these technologies. In Section 3, we define both the traditional output-oriented and the new input-oriented plant capacity notions, after which we argue and illustrate that the output-oriented plant capacity notion may well fail in terms of attainability, although this is not an issue for the input-oriented plant capacity concept. We end this section by defining an attainable output-oriented plant capacity notion that incorporates either firm or industry constraints on the availability of variable inputs. Section 4 describes the secondary data set selected for the empirical illustration and summarizes the empirical results in great detail. Section 5 ends with concluding remarks.

2. Technology: Basic Definitions

This section introduces some basic notation and defines the technology. Given an N-dimensional input vector $x \in \mathbb{R}^N_+$ and an M-dimensional output vector $y \in \mathbb{R}^M_+$, we define the production possibility set or technology T as $T = \{(x, y) \mid x \text{ can produce } y\}$. The input set, which is associated with T, denotes all input vectors x capable of producing a given output vector y: $L(y) = \{x \mid (x, y) \in T\}$. Analogously, the output set associated with T denotes all output vectors y that can be produced from a given input vector x: $P(x) = \{y \mid (x, y) \in T\}$.

Throughout this contribution, technology T satisfies some combination of the following standard assumptions.

Assumption T.1. *There is the possibility of inaction and no free lunch; that is,* $(0, 0) \in T$, *and if* $(0, y) \in T$, *then* $y = 0$.

Assumption T.2. *Technology* T *is a closed subset of* $\mathbb{R}^N_+ \times \mathbb{R}^M_+$.

Assumption T.3. *Strong input and output disposality exists; that is, if* $(x, y) \in T$ *and* $(x', y') \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$, *then* $(x', -y') \geq (x, -y) \Rightarrow (x', y') \in T$.

Assumption T.4. *Technology* T *is convex.*

With regard to these traditional axioms on technology (for details, see Hackman 2008), it is useful to recall that inaction is feasible, there is no free lunch, and technology is closed. We assume free disposal of inputs and outputs in that inputs can be wasted and outputs can be discarded. Finally, technology is convex. In our empirical analysis, not all these axioms are simultaneously maintained.[1] In particular, the key assumption distinguishing some of the technologies in the empirical analysis is convexity versus nonconvexity.

The radial input efficiency measure characterizes the input set $L(y)$ completely and can be defined as

$$DF_i(x, y) = \min\{\lambda \mid \lambda \geq 0, \lambda x \in L(y)\}. \tag{1}$$

This radial input efficiency measure has the main property that it is smaller than or equal to unity ($DF_i(x, y) \leq 1$), with efficient production on the boundary (isoquant) of $L(y)$ represented by unity, and that it has a cost interpretation (see Hackman 2008).

The radial output efficiency measure offers a complete characterization of the output set $P(x)$ and can be defined as

$$DF_o(x, y) = \max\{\theta \mid \theta \geq 0, \theta y \in P(x)\}. \tag{2}$$

Its main properties are that it is larger than or equal to unity ($DF_o(x, y) \geq 1$), with efficient production on the boundary (isoquant) of the output set $P(x)$ represented by unity, and that this radial output efficiency measure has a revenue interpretation (see Hackman 2008).

In the short run, we can partition the input vector into a fixed and variable part. In particular, we denote $(x = (x^f, x^v))$ with $x^f \in \mathbb{R}^{N_f}_+$ and $x^v \in \mathbb{R}^{N_v}_+$, such that $N = N_f + N_v$.

Following Färe et al. (1989b), we can define a short-run technology $T^f = \{(x^f, y) \in \mathbb{R}^{N_f}_+ \times \mathbb{R}^M_+ \mid (x^f, x^v) \text{ can produce } y\}$ and the corresponding input set $L^f(y) = \{x^f \in \mathbb{R}^{N_f}_+ \mid (x^f, y) \in T^f\}$ and output set $P^f(x^f) = \{y \mid (x^f, y) \in T^f\}$. Note that we can obtain technology T^f by projecting technology $T \in \mathbb{R}^{N+M}_+$ into the subspace $\mathbb{R}^{N_f+M}_+$ (i.e., by setting all variable inputs equal to zero). Analogously, the same applies to the input set $L^f(y)$ and the output set $P^f(x^f)$.

Denoting the radial output efficiency measure of the output set $P^f(x^f)$ by $DF^f_o(x^f, y)$, we can define this output-oriented efficiency measure as

$$DF^f_o(x^f, y) = \max\{\theta \mid \theta \geq 0, \theta y \in P^f(x^f)\}. \tag{3}$$

We define the subvector input efficiency measure reducing only the variable inputs as

$$DF^{SR}_i(x^f, x^v, y) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(y)\}. \tag{4}$$

Next, we need the following particular definition of technology: $L(0) = \{x \mid (x, 0) \in T\}$ is the input set with zero output level. The subvector input efficiency measure reducing variable inputs evaluated relative to this input set with a zero output level is

$$DF^{SR}_i(x^f, x^v, 0) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(0)\}. \tag{5}$$

Given data on K observations ($k = 1, \ldots, K$) consisting of a vector of inputs and outputs $(x_k, y_k) \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$, a unified algebraic representation of convex and nonconvex nonparametric frontier technologies under the flexible or variable returns to scale assumption is possible, as follows:

$$T^{\Lambda} = \left\{(x,y) \mid x \geq \sum_{k=1}^{K} z_k x_k, y \leq \sum_{k=1}^{K} z_k y_k, z \in \Lambda\right\}, \quad (6)$$

where

$$\text{(i) } \Lambda \equiv \Lambda^{C} = \left\{z \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \geq 0\right\};$$

$$\text{(ii) } \Lambda \equiv \Lambda^{NC} = \left\{z \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \in \{0,1\}\right\}.$$

The activity vector z of real numbers summing to unity represents the convexity axiom. This same sum constraint with each vector element being a binary integer represents nonconvexity. The convex technology satisfies axioms T.1 (except inaction) to T.4, whereas the nonconvex technology adheres to axioms T.1 to T.3. It is now useful to condition the above notation (6) of the efficiency measures relative to these nonparametric frontier technologies by distinguishing between convexity (convention C) and nonconvexity (convention NC).

A common assumption is that the input and output data satisfy a series of conditions (Färe et al. 1994): (i) each producer employs nonnegative numbers of each input to produce nonnegative numbers of each output, (ii) there is an aggregate production of positive numbers of every output as well as an aggregate utilization of positive numbers of every input, and (iii) each producer employs a positive number of at least one input to produce a positive number of at least one output.

3. Plant Capacity Concepts

3.1. Plant Capacity: Basic Definitions

According to Johansen (1987, p. 362), the informal definition of plant capacity is "the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." In turn, Färe et al. (1989a, b) operationalize the output-oriented plant capacity notion using a pair of output-oriented efficiency measures. We can now define this output-oriented plant capacity utilization (PCU) as follows.

Definition 1. The output-oriented plant capacity utilization (PCU_o) is

$$PCU_o(x, x^f, y) = \frac{DF_o(x,y)}{DF_o^f(x^f, y)},$$

where $DF_o(x,y)$ and $DF_o^f(x^f,y)$ are output efficiency measures that, respectively, include and exclude the variable inputs as defined in (2) and (3). Because $1 \leq DF_o(x,y) \leq DF_o^f(x^f,y)$, $0 < PCU_o(x,x^f,y) \leq 1$. Thus, output-oriented plant capacity utilization has an upper limit of unity. Following the terminology introduced by Färe et al. (1989a, b) and Färe et al. (1994), we can distinguish between a so-called biased plant capacity measure $DF_o^f(x^f,y)$ and an unbiased plant capacity measure $PCU_o(x,x^f,y)$. Taking the ratio of efficiency measures eliminates any existing inefficiency and yields in this sense a cleaned concept of output-oriented plant capacity.

To guarantee the existence of the efficiency measures, Färe et al. (1989a, pp. 659–660) sharpen the conditions on the input and output data for nonparametric frontier technologies.[2] In particular, each fixed input is used by some producer, and each producer uses some fixed input.

In the case of C, the efficiency measure $DF_o^f(x^f,y)$ is computed for observation (x_p, y_p) as follows:

$$\begin{aligned} DF_o^f(x_p^f, y_p) = \max_{\theta, z_k}\ & \theta \\ \text{s.t. } & \sum_{k=1}^{K} z_k y_k \geq \theta y_p, \\ & \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\ & \sum_{k=1}^{K} z_k = 1, \\ & \theta \geq 0, z_k \geq 0, \quad k = 1, \ldots, K. \end{aligned} \quad (7)$$

In the case of NC, the variables z_k in this model need to be binary variables. In all linear programming (LP) models mentioned hereafter, a similar adaptation is required if NC is assumed. To save space, we no longer mention this or formulate the corresponding models.

Note that there are no input constraints on the variable inputs in the model (7). Note that Färe et al. (1994) introduce an alternative LP with a scalar for each variable input dimension. In addition, LP (7) is equivalent to the following LP obtained by making each variable input a decision variable:

$$\begin{aligned} DF_o^f(x_p^f, y_p) = \max_{\theta, z_k, x^v}\ & \theta \\ \text{s.t. } & \sum_{k=1}^{K} z_k y_k \geq \theta y_p, \\ & \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\ & \sum_{k=1}^{K} z_k x_k^v \leq x^v, \\ & \sum_{k=1}^{K} z_k = 1, \\ & \theta \geq 0, z_k \geq 0, x^v \geq 0, \ k = 1, \ldots, K. \end{aligned} \quad (8)$$

Cesaroni et al. (2017) define a new input-oriented plant capacity measure using a pair of input-oriented efficiency measures.

Definition 2. The input-oriented plant capacity utilization (PCU_i) is

$$PCU_i(x, x^f, y) = \frac{DF_i^{SR}(x^f, x^v, y)}{DF_i^{SR}(x^f, x^v, 0)},$$

where $DF_i^{SR}(x^f, x^v, y)$ and $DF_i^{SR}(x^f, x^v, 0)$ are both sub-vector input efficiency measures reducing only the variable inputs relative to the technology, where the latter efficiency measure is evaluated at a zero output level. Because $0 < DF_i^{SR}(x^f, x^v, 0) \leq DF_i^{SR}(x^f, x^v, y)$, $PCU_i \cdot (x, x^f, y) \geq 1$. Thus, input-oriented plant capacity utilization has a lower limit of unity. Similar to the previous case, we can distinguish between a so-called biased plant capacity measure $DF_i^{SR}(x^f, x^v, 0)$ and an unbiased plant capacity measure $PCU_i^{SR}(x, x^f, y)$, the latter of which is cleaned of any prevailing inefficiency.[3]

To guarantee the existence of the efficiency measures, we also need to sharpen the conditions on the input and output data: each variable input is used by some producer, and each producer uses some variable input.

Both Definitions 1 and 2 are graphically illustrated with the help of Figure EC1 in Section EC.1 of the e-companion. We now turn to the issue of attainability of both these plant capacity concepts.

3.2. Plant Capacity: The Question About Attainability

Although Definitions 1 and 2 are sufficiently clear, we stress that these concepts differ in terms of the property of attainability. According to Johansen (1987), the output-oriented plant capacity notion is not attainable if the extra variable inputs necessary to reach the maximal plant capacity output are not available. Whereas in principle the axiom of strong disposability in the inputs allows for the wasting of infinitely many inputs to determine the maximal plant capacity outputs, in practice there may well be various restrictions that limit the availability of variable inputs.[4]

First, at the firm level there may be quasi-fixed factors such as labor for which firms have to invest in hiring and training activities that limit the number of people who can be recruited at once. By definition, quasi-fixed factors are characterized by the inability to expand their supply rapidly. Furthermore, depending on the nature of the labor market and the size of the firm (e.g., it may have some monopsony power), recruiting a large number of people may well have an impact on their salaries. Although this does not show up in the analytical framework of the output-oriented plant capacity notion, which ignores input prices, firms may well account for these general equilibrium effects and constrain their recruitment of the quasi-fixed factor. In brief, the quasi-fixity of labor as well as other production factors may seriously impede the expansion of variable inputs and thus may prevent firms from reaching the maximal plant capacity outputs (see Oi 1962 for the seminal article in economics and Barney 2001 for the resource-based view of the firm).

Second, even if these extra variable inputs are available at the firm level, restrictions on the available extra variable inputs at the sector level may prevent all firms from simultaneously reaching their maximal plant capacity output (Johansen 1987). For example, quasi-fixed factors may operate at the industry level and prevent the rapid expansion of supply in amounts necessary to realize the maximal plant capacity outputs for all firms. At the sectoral level, general equilibrium effects may play a role: if all firms simultaneously increase their demand for a production factor, then the price of that production factor may well increase. Again, although this does not show up in the framework of the output-oriented plant capacity notion, which ignores factor prices, firms may take these general equilibrium effects into account and constrain their expansion of the production factor.

By contrast, the input-oriented plant capacity notion is always attainable in that firms can always reduce the number of existing variable inputs to reach an input set with a zero output level. Reducing variable inputs to reach zero production levels is normally possible because of the axiom of inaction. Inaction implies that firms can stop producing in full: but in modern production facilities, producing a zero output does not necessarily imply that no inputs are used.[5] Examples of zero production with positive numbers of variable inputs include critical maintenance activities at a large industrial plant impeding production, counting inventory in a retailer while temporarily suspending sales, or temporarily closing a mine while keeping it exploitable with the option of reopening it as part of a real options strategy. Closing down production is therefore possible at the firm level, but it can also be done at the sectoral level.

Therefore, attainability is a potential issue for the output-oriented plant capacity notion, whereas it is a priori not an issue for the new input-oriented plant capacity concept. We now turn to the modeling of constraints on the availability of variable inputs in the output-oriented plant capacity notion.

A somewhat related issue is the economic relevance of these plant capacity notions. Again considering the output-oriented plant capacity concept, even if the firm has sufficiently variable inputs at its disposal and the attainability issue does not exist, producing the output-oriented plant capacity outputs will rarely be cost minimizing or profit maximizing. This technical or engineering capacity concept just serves as a generalization of other popular capacity concepts (e.g., in the hotel industry, room occupancy rates are very popular) for multiple-output production processes. For the case of the input-oriented plant capacity concept, for which the attainability issue does not exist, the question as to the relevance of the optimal variable inputs at the level of the initialization of production is also important. As previously mentioned, maintenance activities may lead to temporarily suspending production, as well as temporarily mothballing operations. However, for most firms, these optimal variable inputs at zero output levels may not follow from a cost-minimizing or profit-maximizing strategy. Again, this technical or engineering capacity concept just serves as a framework to summarize capacity measurement for multiple inputs and outputs production processes.

Cesaroni et al. (2019) also recently defined new long-run output- and input-oriented plant capacity concepts that allow for changes in all input dimensions simultaneously rather than changes in the variable inputs only. The plant capacity concepts focusing on changes in the variable inputs alone can then be interpreted as short-run concepts. Note that the whole issue of attainability also transposes to the output- and input-oriented long-run plant capacity concepts.

3.3. Attainable Output-Oriented Plant Capacity: Proposals

We now turn to the specification of attainability constraints at the firm level. Thereafter, we explore how to model attainability constraints at the industry level.

3.3.1. Attainability Constraints at the Firm Level. The standard assumption T.3 of strong input and output disposability implies that variable inputs can be increased without limitation in the absence of price information. However, with the reality of limited resources, we know that this possibility of allowing unlimited increases of inputs creates a potential issue. This issue also affects all notions built on this possibility, especially the output-oriented plant capacity notion. As a possible remedy, we define an attainability level $\bar{\lambda}$ of an observation as follows.

Definition 3. An attainability level $\bar{\lambda}$ of observation (x_p, y_p) (abbreviated to level $\bar{\lambda}$) is any value $\bar{\lambda} \in \mathbb{R}_+$ satisfying

$$\exists \lambda \in \mathbb{R}_+ \text{ with } \lambda \le \bar{\lambda} \text{ and}$$
$$\exists \theta \in \mathbb{R}_+ \text{ such that } (x^f, \lambda x^v, \theta y) \in T.$$

It follows from this definition that every value $\bar{\lambda} \ge 1$ can serve as an attainability level for all observations (e.g., set $\lambda = 1$ and $\theta = 1$). An attainability level $\bar{\lambda} < 1$ might not be possible for some observations (see the empirical illustration in Section 4). However, this level should be chosen to reflect a realistic achievable upscaling of variable inputs for a particular observation. For example, a value $\bar{\lambda} = 3$ means that tripling variable inputs can be realistic (or achievable).

Note that Definition 3 differs from the rather well-known axiom of attainability as developed by Shephard (see Färe and Mitchell 1987 for a critical discussion).

With an attainability level set to some realistic value, we can define the following attainable output-oriented efficiency measure.

Definition 4. The attainable output-oriented efficiency measure (ADF_o) at level $\bar{\lambda} \in \mathbb{R}_+$ is

$$ADF_o^f(x^f, y, \bar{\lambda}) = \max\{\theta \mid \theta \ge 0, 0 \le \lambda \le \bar{\lambda}, \theta y \in P(x^f, \lambda x^v)\}.$$

The number of variable inputs is now bound to be at most a scalar-wise multiple smaller than $\bar{\lambda}$. Then, $ADF_o^f \cdot (x^f, y, \bar{\lambda}) \le DF_o^f(x^f, y)$. Note that we write Definition 4 in absolute terms. For example, $\bar{\lambda} = 3$ corresponds to the impossibility of variable inputs exceeding three times the current amount of variable inputs. Alternatively, we could focus on relative comparisons with the sector aggregates $(\sum_{p=1}^{K} x_p^v)$. Here, we could impose the constraint that variable inputs at the firm level cannot exceed a certain share of the total number of variable inputs available in a sector. We opt for the former approach.

Using the attainable output-oriented efficiency measure introduced in Definition 4, we can define a new attainable output-oriented plant capacity concept at the firm level.

Definition 5. An attainable output-oriented plant capacity utilization $(APCU_o)$ at level $\bar{\lambda} \in \mathbb{R}_+$ is

$$APCU_o(x, x^f, y, \bar{\lambda}) = \frac{DF_o(x, y)}{ADF_o^f(x^f, y, \bar{\lambda})},$$

with $DF_o(x, y)$ and $ADF_o^f(x^f, y, \bar{\lambda})$ as defined previously.

Analogous to the plant capacity utilization measures introduced in Definitions 1 and 2, we can distinguish between the attainable biased plant capacity measure $ADF_o^f(x^f, y, \bar{\lambda})$ and the attainable unbiased plant capacity measure $APCU_o(x, x^f, y, \bar{\lambda})$, where the ratio of efficiency measures ensures the elimination of any existing inefficiency.

Because $ADF_o^f(x^f, y, \bar{\lambda}) \le DF_o^f(x^f, y)$, $APCU_o(x, x^f, y, \bar{\lambda}) \ge PCU_o(x, x^f, y)$. Thus, the attainable output-oriented measure of plant capacity utilization is always larger than or equal to the traditional measure of output-oriented plant capacity utilization.

Proposition 1. *The attainable output-oriented plant capacity utilization $APCU_o(x, x^f, y, \bar{\lambda})$ converges to the output-oriented plant capacity utilization $PCU_o(x, x^f, y)$ as $\bar{\lambda} \longrightarrow \infty$ (i.e., $\lim_{\bar{\lambda} \to \infty} APCU_o(x, x^f, y, \bar{\lambda}) = PCU_o(x, x^f, y)$).*

For the proofs of all propositions, see Section EC.2 in the e-companion.

Note also that the output-oriented plant capacity utilization $PCU_o(x, x^f, y)$ might be unrealistic because the numbers of variable inputs required to reach the maximum capacity outputs may simply not be available. This can be observed in the empirical illustration in Section 4. Thus, $APCU_o(x, x^f, y, \bar{\lambda})$ should be a more realistic alternative plant capacity utilization measure provided an achievable level $\bar{\lambda}$ is chosen.

We can now model the attainability constraints at the firm level as follows:

$$\begin{aligned} ADF_o^f(x_p^f, y_p, \bar{\lambda}) = \max_{\theta, z_k, x^v} \theta & \\ \text{s.t.} \sum_{k=1}^{K} z_k y_k &\geq \theta y_p, \\ \sum_{k=1}^{K} z_k x_k^f &\leq x_p^f, \\ \sum_{k=1}^{K} z_k x_k^v &\leq x^v, \\ \sum_{k=1}^{K} z_k &= 1, \\ x^v &\leq \bar{\lambda} x_p^v, \\ \theta \geq 0, z_k \geq 0, x^v &\geq 0, \; k = 1, \ldots, K. \end{aligned} \tag{9}$$

The constraint $x^v \leq \bar{\lambda} x_p^v$ establishes a link between the decision variable x^v and the value x_p^v of the firm under observation. In the empirical analysis of Section 4, we choose $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$. Thus, we regard an increase of the variable inputs with a factor more than 5 or less than 0.5 (i.e., halving these variable inputs) as implausible.

In model (9), we can vary the scalar $\bar{\lambda}$ over some part of the interval $(0, \infty)$. To determine the complete feasible interval for $\bar{\lambda}$ and to classify $ADF_o^f(x^f, y, \lambda)$ and $APCU_o(x, x^f, y, \lambda)$ subsequently, we need the following definition of critical points.

Definition 6. For a given observation (x_p, y_p), we can define the following three critical points L_p, M_p, and U_p:

$$L_p = DF_i^{SR}(x_p^f, x_p^v, 0), \tag{10}$$

$$M_p = DF_i^{SR}(x_p^f, x_p^v, y_p), \tag{11}$$

and

$$U_p = DF_i^{SR}(x_p^f, x_p^v, DF_o^f(x_p^f, y_p) y_p). \tag{12}$$

Note that the critical points L_p and M_p make up the components of the input-oriented plant capacity measure $PCU_i(x, x^f, y)$ in Definition 2. To our knowledge, U_p has not been described previously in the literature. It can be interpreted as the minimal expansion of variable inputs needed to produce the maximum plant capacity outputs and can be computed as

$$\begin{aligned} U_p = \min_{\theta, z_k} \theta & \\ \text{s.t.} \sum_{k=1}^{K} z_k y_k &\geq DF_o^f(x_p^f, y_p) y_p, \\ \sum_{k=1}^{K} z_k x_k^f &\leq x_p^f, \\ \sum_{k=1}^{K} z_k x_k^v &\leq \theta x_p^v, \\ \sum_{k=1}^{K} z_k &= 1, \\ \theta \geq 0, z_k &\geq 0, k = 1, \ldots, K. \end{aligned} \tag{13}$$

We briefly illustrate these three critical points in Figure EC1. First, the point L_p pertains to the distance from point a to point e'': it indicates the minimal number of variable inputs compatible with zero outputs. Second, the point M_p pertains to the distance from point e'''' to point e: it indicates the minimal number of variable inputs compatible with current levels of outputs. Third, the point U_p pertains to the distance from point e to point e': it indicates the minimal number with which variable inputs need to be expanded to be compatible with the maximal level of plant capacity outputs at point d.

We are now in a position to classify $ADF_o^f(x^f, y, \bar{\lambda})$ and $APCU_o(x, x^f, y, \bar{\lambda})$ in terms of these three critical points. In particular, we establish two propositions.

Proposition 2. *For the attainable biased and unbiased output-oriented plant capacity utilization in both C and NC technologies, for every observation* (x_p, y_p),

(i) *if* $\bar{\lambda} < L_p$, *then model* (9) *is infeasible;*

(ii) *if* $L_p \leq \bar{\lambda} < M_p$, *then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < 1$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > 1$; *and*

(iii) *if* $M_p \leq \bar{\lambda}$, *then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq 1$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) \leq 1$.

Proposition 3. *For the attainable biased and unbiased output-oriented plant capacity utilization in both C and NC technologies, for every observation* (x_p, y_p),

(i) *if* $L_p \leq \bar{\lambda} < U_p$, *then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < DF_o^f(x_p^f, y_p)$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > PCU_o(x_p, x_p^f, y_p)$; *and*

(ii) *if* $\bar{\lambda} \geq U_p$, *then* $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ *and* $APCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$.

3.3.2. Attainability Constraints at the Industry Level.

Similar to the firm-level situation, it is also feasible to devise new attainable output-oriented plant capacity concepts at the industry level. First, we introduce the industry-attainable output-oriented efficiency measure as follows.

Definition 7. The industry-attainable output-oriented efficiency measure ($IADF_o$) at level $\bar{\lambda} \in \mathbb{R}_+$ for observation (x_p, y_p) is

$$IADF_o^f(x_p^f, y_p, \bar{\lambda}) = \theta_p^*,$$

where θ_p^* is the optimum value of θ_p in the following model:

$$\begin{aligned}
\max_{\theta_p, z_k^p, x_p^v} \quad & \sum_{p=1}^{K} \theta_p \\
\text{s.t.} \quad & \sum_{k=1}^{K} z_k^p y_k \geq \theta_p y_p, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p x_k^f \leq x_p^f, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p x_k^v \leq x_p^v, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p = 1, && p = 1, \ldots, K, \\
& \sum_{p=1}^{K} x_p^v \leq \bar{\lambda} \sum_{p=1}^{K} \bar{x}_p^v, \\
& \theta_p \geq 0, z_k^p \geq 0, x_p^v \geq 0, \ k, p = 1, \ldots, K.
\end{aligned} \tag{14}$$

Note that model (14) is a kind of central resource allocation model with K LPs (one for each observation) and a bogus objective function and with a common constraint on the total number of variable inputs available in the sector. In particular, its aim is to determine the maximum plant capacity outputs for all observations while reallocating variable inputs among units, such that a global constraint on the industry number of variable inputs is respected. Central resource reallocation models cover a heterogeneous variety of models reallocating some inputs and/or outputs across space and/or time while eventually accounting for multiple objectives (e.g., efficiency, effectiveness, equality). Athanassopoulos (1998), Färe et al. (1992), Golany and Tamir (1995), Korhonen and Syrjänen (2004), Lozano and Villa (2004), and Ylvinger (2000) provide examples of these models. One type of central resource reallocation model that also makes use of the notion of plant capacity is the so-called short-run Johansen industry model (see Färe et al. 1992 for a single-output version and Kerstens et al. 2006 for a multiple-output version). Note that if we remove the last constraint in model (14), then we obtain the industry output-oriented efficiency measure (IDF_o) for observation (x_p, y_p): $IDF_o^f(x_p^f, y_p) = \theta_p^{**} = DF_o^f(x_p^f, y_p)$, where θ_p^{**} is the optimal value of θ_p in model (14) without its last constraint. Thus, this new industry output-oriented efficiency measure (IDF_o) coincides with the firm output-oriented efficiency measure (DF_o). Note also that—to the best of our knowledge—this contribution is the first to discuss plant capacity at the industry level.

Second, using the industry-attainable output-oriented efficiency measure of Definition 7, we can define the industry-attainable output-oriented plant capacity utilization as follows.

Definition 8. The industry-attainable output-oriented plant capacity utilization ($IAPCU_o$) at level $\bar{\lambda} \in \mathbb{R}_+$ for observation (x_p, y_p) is

$$IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = \frac{DF_o(x_p, y_p)}{IADF_o^f(x_p^f, y_p, \bar{\lambda})}.$$

Because $IADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o^f(x^f, y)$, $IAPCU_o(x, x^f, y, \bar{\lambda}) \geq PCU_o(x, x^f, y)$. Thus, the industry-attainable output-oriented measure of plant capacity utilization is always larger than or equal to the traditional measure of output-oriented plant capacity utilization. Analogously, we can distinguish between the industry-attainable biased plant capacity measure $IADF_o^f(x^f, y, \bar{\lambda})$ and the industry-attainable unbiased plant capacity measure $IAPCU_o(x, x^f, y, \bar{\lambda})$, where the ratio of efficiency measures ensures elimination of any existing inefficiency.

Note that the industry-attainable output-oriented measure of plant capacity utilization may be smaller or larger than the attainable output-oriented measure of plant capacity utilization. This holds true for both the biased and unbiased versions. Therefore, we have $IADF_o^f(x^f, y, \bar{\lambda}) \gtreqless ADF_o^f(x_p^f, y_p, \bar{\lambda})$ and $IAPCU_o(x, x^f, y, \bar{\lambda}) \gtreqless APCU_o(x, x^f, y, \bar{\lambda})$.

Analogous to firm-level modeling, we can vary the scalar $\bar{\lambda}$ in model (14) over some part of the interval $(0, \infty)$. To determine this feasible interval for $\bar{\lambda}$, we can define the following two critical points L^I and U^I.

Definition 9. L^I can be determined from the following LP:

$$\begin{aligned}
L^I = \min_{\theta, z_k^p, x_p^v} \ & \theta \\
\text{s.t.} \quad & \sum_{k=1}^{K} z_k^p x_k^f \leq x_p^f, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p x_k^v \leq x_p^v, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p = 1, && p = 1, \ldots, K, \\
& \sum_{p=1}^{K} x_p^v \leq \theta \sum_{p=1}^{K} \bar{x}_p^v, \\
& \theta \geq 0, z_k^p \geq 0, x_p^v \geq 0, \ k, p = 1, \ldots, K.
\end{aligned} \tag{15}$$

We obtain U^I by solving the following LP:

$$
\begin{aligned}
U^I = \min_{\theta, z_k^p, x_p^v} \ & \theta \\
\text{s.t.} \ & \sum_{k=1}^{K} z_k^p y_k \geq DF_o^f(x_p^f, y_p) y_p, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p x_k^f \leq x_p^f, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p x_k^v \leq x_p^v, && p = 1, \ldots, K, \\
& \sum_{k=1}^{K} z_k^p = 1, && p = 1, \ldots, K, \\
& \sum_{p=1}^{K} x_p^v \leq \theta \sum_{p=1}^{K} \bar{x}_p^v, \\
& \theta \geq 0, z_k^p \geq 0, x_p^v \geq 0, && k, p = 1, \ldots, K.
\end{aligned} \tag{16}
$$

Note that U^I can be interpreted as the minimal expansion of overall variable inputs needed to produce the plant capacity outputs for all units for the industry model (14).

We are now in a position to classify $IADF_o^f(x^f, y, \bar{\lambda})$ and $IAPCU_o(x, x^f, y, \bar{\lambda})$ in terms of these two critical points.

Proposition 4. *For the industry-attainable biased and unbiased output-oriented plant capacity utilization in both C and NC technologies,*

(i) *if $\bar{\lambda} < L^I$, then model (14) is infeasible;*

(ii) *if $L^I \leq \bar{\lambda} < U^I$, then at least for one observed observation (x_p, y_p), we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) < DF_o^f(x_p^f, y_p)$ and $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) > PCU_o(x_p, x_p^f, y_p)$; and*

(iii) *if $U^I \leq \bar{\lambda}$, then for every observation (x_p, y_p), we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ and $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$.*

4. Empirical Illustration

4.1. Description of the Sample

For the empirical illustration of the attainability notions introduced previously, we use a secondary data set from Atkinson and Dorfman (2009). The sample is based on 16 Chilean hydroelectric power generation plants observed on a monthly basis. We limit our observations to the year 1997, and assuming that there is no technical change, we specify an intertemporal frontier across all 12 months, which results in 192 units. It is well known that Chile was one of the first countries to deregulate its electricity market and that hydropower was a dominant source of energy during the 1990s. These hydropower plants generate one output (electricity) using three inputs: labor, capital, and water. Except for the fixed input capital, we express the remaining flow variables in physical units. Table 1 presents basic descriptive statistics for the inputs and the single output. It shows a large heterogeneity in terms of size among the different inputs and the single output.

4.2. Empirical Results for Firm Level

We structure Tables 2 and 3 in a similar way. Table 2 reports the biased plant capacity utilization measures $DF_o^f(x^f, y)$ and $ADF_o^f(x^f, y, \bar{\lambda})$, and Table 3 focuses on the unbiased plant capacity utilization measures $PCU_o(x, x^f, y)$ and $APCU_o(x, x^f, y, \bar{\lambda})$. In each table, the second column reports the standard plant capacity utilization measures, whereas the next 10 columns describe the attainable plant capacity utilization measures for $\bar{\lambda}$ varying between 0.5 and 5 with step size 0.5 (thus, $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$). Therefore, we somewhat arbitrarily assume that variable inputs can be magnified at most fivefold. Note that we could have selected a wider range of values to experiment with $\bar{\lambda}$. In line with Proposition 2, for 37 observations under C and 41 observations under NC, $\bar{\lambda} = 0.5$ is too small for model (9) to be feasible. Therefore, we do not include these observations in the corresponding descriptive statistics computations.

Our data set contains a single output, which implies that $DF_o^f(x_p^f, y_p)$ is homogeneous of degree −1 in the output. From (12), it then follows that $U_p = DF_i^{SR}(x_p^f, x_p^v, DF_o^f(x_p^f, 1))$. Then, U_p now depends only on the variable and fixed inputs. However, in the multioutput case, this observation no longer holds true. For an example with multiple outputs, see Section EC.3 in the e-companion.

Analyzing the results in Table 2, we can draw the following conclusions. First, the biased plant capacity utilization measure $DF_o^f(x^f, y)$ indicates that outputs can be magnified by at least 13.65 times under C and

Table 1. Descriptive Statistics for Hydropower Plants (1997)

Variable	Trimmed mean[a]	Minimum	Maximum
Billions of cubic meters of water (variable input)	126.80	0.49	1,347.47
No. of workers (variable input)	15.62	2.00	52.86
Billions of capital (fixed input)	0.47	0.04	5.98
Thousands of kilowatt-hours (output)	46.79	0.40	353.70

[a]There is a 10% trimming level.

Table 2. Descriptive Statistics of Biased Plant Capacity Utilization

		$ADF_o^f(x^f, y, \bar{\lambda})$									
	$DF_o^f(x^f, y)$	$\bar{\lambda} = 0.5$	$\bar{\lambda} = 1$	$\bar{\lambda} = 1.5$	$\bar{\lambda} = 2$	$\bar{\lambda} = 2.5$	$\bar{\lambda} = 3$	$\bar{\lambda} = 3.5$	$\bar{\lambda} = 4$	$\bar{\lambda} = 4.5$	$\bar{\lambda} = 5$
Convex											
Average	13.655	1.017	1.663	2.191	2.594	2.912	3.153	3.358	3.547	3.721	3.877
SD	77.137	1.027	1.721	2.421	3.107	3.770	4.349	4.927	5.502	6.077	6.645
Minimum	1.000	0.252	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Maximum	884.250	7.732	15.465	21.695	27.937	34.322	38.807	43.290	47.775	52.259	56.743
Nonconvex											
Average	12.541	0.600	1.275	1.508	1.746	1.942	2.166	2.367	2.547	2.701	2.762
SD	77.226	0.907	1.511	1.818	2.157	2.830	2.856	3.501	4.303	4.725	5.079
Minimum	1.000	0.118	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Maximum	884.250	7.714	13.500	19.000	21.000	33.250	33.250	33.250	40.286	43.714	45.500

12.54 times under NC, on average. Second, there is a great amount of variation in $DF_o^f(x^f, y)$, as indicated by the standard deviation, and the range is broad: the maximum increase in outputs amounts to 884.25 times under both C and NC. Third, the attainable biased plant capacity utilization measure $ADF_o^f(x^f, y, \bar{\lambda})$ increases monotonically in $\bar{\lambda}$, and on average, the output magnification under C is always higher than that under NC. Fourth, for a fivefold increase in variable inputs (i.e., $\bar{\lambda} = 5$), we obtain on average a 3.87 output magnification under C and a 2.76 output magnification under NC. This is quite a bit lower than the average output magnification computed by the biased plant capacity utilization measure $DF_o^f(x^f, y)$.

Turning to the analysis of Table 3, we can draw several conclusions. First, the unbiased plant capacity utilization measure $PCU_o(x^f, y)$ indicates that current outputs make up 52% from maximal plant capacity outputs under C and 55% under NC, on average. Second, there is a great amount of heterogeneity in $PCU_o(x^f, y)$, as indicated by the standard deviation, and the range is again broad: the minimum of approximately 1.5% under both C and NC is simply extremely low. Third, the attainable unbiased plant capacity utilization measure $APCU_o(x, x^f, y, \bar{\lambda})$ decreases monotonically in $\bar{\lambda}$, and on average, $APCU_o(x, x^f, y, \bar{\lambda})$ is always smaller under C than under NC. Fourth, for a fivefold increase in variable inputs (i.e., $\bar{\lambda} = 5$), $APCU_o(x, x^f, y, \bar{\lambda})$ moves close to $PCU_o(x, x^f, y)$ in the C case (a difference of only 4%), whereas this gap is larger in the NC case (a difference of 8%).

Table 4 reports descriptive statistics on the three critical points L_p, M_p, and U_p as defined in Definition 6. Our inferred conclusions are fourfold. First, the average values for L_p and M_p are rather moderate, and the values are lower under C than under NC each time. This leads to rather plausible results for the input-oriented plant capacity measure $PCU_i(x, x^f, y)$. Under C, an average of 4.39 more variable inputs with current outputs than with zero outputs are required, whereas under NC, 6.21 more variable inputs with current outputs than with zero outputs are required.

Second, on average, the critical point U_p is very high: 31.58 times more variable inputs than currently in use are required to reach maximum plant capacity outputs under C, whereas magnifying variable inputs by just a factor of 28.75 would be necessary under NC. These amounts are much larger than our prior value of allowing for only a fivefold increase in variable inputs.

Third, the variation in this factor U_p is also large. For example, at the third quartile, we obtain a 12.29 magnification factor under C and only a 5.44 magnification

Table 3. Descriptive Statistics of Unbiased Plant Capacity Utilization

		$APCU_o(x, x^f, y, \bar{\lambda})$									
	$PCU_o(x, x^f, y)$	$\bar{\lambda} = 0.5$	$\bar{\lambda} = 1$	$\bar{\lambda} = 1.5$	$\bar{\lambda} = 2$	$\bar{\lambda} = 2.5$	$\bar{\lambda} = 3$	$\bar{\lambda} = 3.5$	$\bar{\lambda} = 4$	$\bar{\lambda} = 4.5$	$\bar{\lambda} = 5$
Convex											
Average	0.522	1.952	1.000	0.778	0.687	0.637	0.611	0.594	0.581	0.572	0.564
SD	0.269	0.705	0.000	0.113	0.156	0.179	0.193	0.203	0.211	0.218	0.223
Minimum	0.016	1.000	1.000	0.495	0.331	0.272	0.231	0.200	0.177	0.159	0.155
Maximum	1.000	4.916	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Nonconvex											
Average	0.553	2.964	1.000	0.868	0.782	0.733	0.677	0.658	0.647	0.634	0.631
SD	0.304	1.424	0.000	0.146	0.187	0.205	0.236	0.244	0.251	0.258	0.260
Minimum	0.015	1.000	1.000	0.459	0.392	0.387	0.259	0.099	0.099	0.099	0.099
Maximum	1.000	8.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4. Descriptive Statistics for Three Critical Points

	Convex				Nonconvex				
	L_p^C	M_p^C	U_p^C	$PCU_i(.)$	L_p^{NC}	M_p^{NC}	U_p^{NC}	$PCU_i(.)$	$U_p^C - U_p^{NC}$
Average	0.338	0.715	31.585	4.397	0.352	0.944	28.753	6.214	2.832
SD	0.301	0.256	106.385	4.877	0.314	0.164	106.372	6.335	4.895
Minimum	0.038	0.132	1.000	1.000	0.038	0.267	0.904	1.000	0.000
First quartile	0.113	0.557	2.628	1.272	0.121	1.000	1.283	1.995	0.000
Median	0.200	0.754	4.031	2.485	0.245	1.000	2.627	3.566	0.571
Third quartile	0.451	0.952	12.295	5.732	0.451	1.000	5.444	7.359	2.692
Maximum	1.000	1.000	648.998	26.070	1.000	1.000	643.500	26.428	25.759

factor under NC. The maximal magnification factors of 648.99 and 643.50 under C and NC, respectively, are similar in magnitude, and both are clearly impossible in reality. These extreme requirements on the availability of variable inputs cast doubt on the plausibility of the traditional output-oriented plant capacity measure.

Fourth, the last column reporting the difference $U_p^C - U_p^{NC}$ reveals that, on average, the variable inputs under C should be increasing at least 2.83 times more than under NC. Furthermore, there is a great amount of heterogeneity in this difference $U_p^C - U_p^{NC}$. Thus, in short, although these magnification factors for the variable inputs are clearly implausible, the nonconvex results are the least implausible.

We end this analysis with the results for certain individual observations. Figures 1 and 2 have two parts: the left-hand side displays the attainable biased plant capacity in function of the value of $\bar{\lambda}$, and the right-hand side shows the attainable unbiased plant capacity in function of the value of $\bar{\lambda}$. We draw both figures under the C and NC assumptions. Furthermore, we draw the same critical point U_p for both C and NC in both figures.

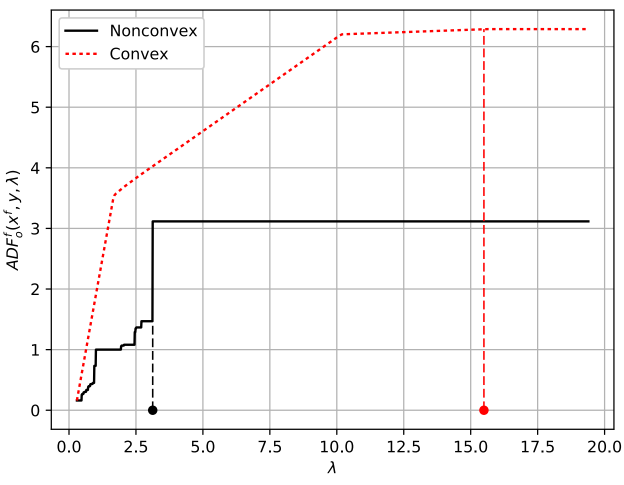
Figure 1 shows the results for plant number 9. With regard to the left-hand side, first the attainable biased plant capacity increases monotonically with $\bar{\lambda}$ under C and in a stepwise fashion under NC: these steps reveal the pervasive problem of slacks that occur under NC. Second, the maximum increase in outputs (i.e., the vertical distance between both lines) for the attainable biased plant capacity is almost double under C than under NC. Third, the value of U_p is almost four times larger under C (15.48) than under NC (3.11). With regard to the right-hand side of Figure 1, first the attainable unbiased plant capacity decreases again monotonically with $\bar{\lambda}$ under C and in a stepwise fashion under NC. Second, the attainable unbiased plant capacity under C crosses with that under NC: only for very high values of $\bar{\lambda}$ are both estimates close to each other. Overall, this again confirms that the NC results are less implausible.

Figure 2 depicts the results for plant number 105. Here, the value of U_p under C and NC is identical (12.82). In this case, the differences between C and NC attainable biased plant capacity are rather pronounced, whereas these differences are mainly visible for the low-range values of $\bar{\lambda}$ for the attainable unbiased plant capacity.

4.3. Empirical Results for Industry Level

We structure Tables 5 and 6 in a similar way to the corresponding firm-level tables. Table 5 reports on the industry-biased plant capacity utilization measure

Figure 1. (Color online) Attainable Biased and Unbiased Plant Capacity for Plant 9



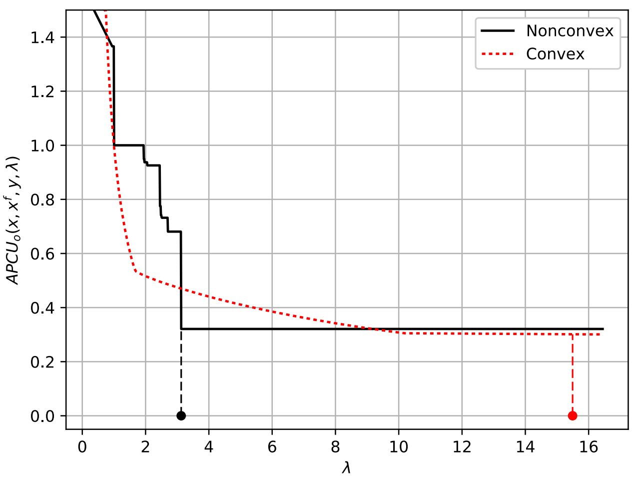
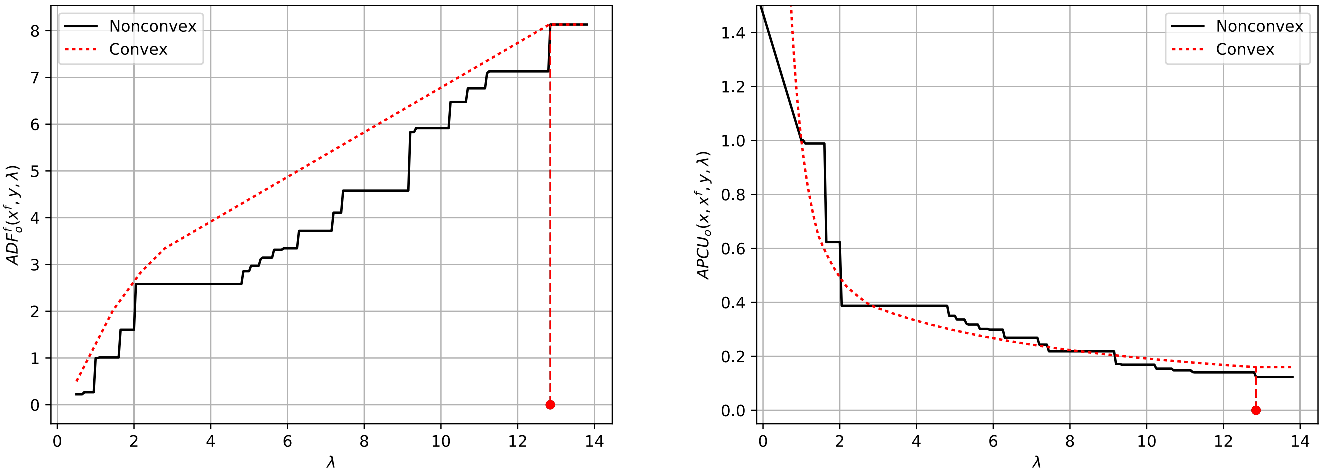


Figure 2. (Color online) Attainable Biased and Unbiased Plant Capacity for Plant 105



$IADF_o^f(x^f, y, \bar{\lambda})$, and Table 6 focuses on the industry-unbiased plant capacity utilization measures $IAPCU_o(x, x^f, y, \bar{\lambda})$. Again, we have 10 columns describing the industry-attainable plant capacity utilization measures for $\bar{\lambda}$ varying between 0.5 and 5 with step size 0.5. The three last rows of Tables 5 and 6 are new and show the number of observed units that have the amounts $ADF_o^f(.) < IADF_o^f(.)$, $ADF_o^f(.) = IADF_o^f(.)$, and $ADF_o^f(.) > IADF_o^f(.)$, respectively, and we focus on comparing firm-level and industry-level results.

In analyzing these results in Table 5, we can draw several conclusions. First, the industry-attainable biased plant capacity utilization measure $IADF_o^f(x^f, y, \bar{\lambda})$ increases almost monotonically in $\bar{\lambda}$, and on average, the output magnification is always higher under C than under NC. Second, $IADF_o^f(x^f, y, \bar{\lambda})$ becomes stationary after $\bar{\lambda}$ reaches the value of 3 under C and the value of 2 under NC. Third, whereas $IADF_o^f(x^f, y, \bar{\lambda}) \gtreqless ADF_o^f(x_p^f, y_p, \bar{\lambda})$, for the majority of observations, we find that $ADF_o^f(x_p^f, y_p, \bar{\lambda}) < IADF_o^f(x^f, y, \bar{\lambda})$ until $\bar{\lambda}$ reaches the value of 4 under C and only 2.5 under NC and $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = IADF_o^f(x^f, y, \bar{\lambda})$ afterward for the majority of observations. Furthermore, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) > IADF_o^f(x^f, y, \bar{\lambda})$ becomes 0 when $IADF_o^f(x^f, y, \bar{\lambda})$ becomes stationary.

Three deductions emerge with regard to the results in Table 6. First, the industry-attainable unbiased plant capacity utilization measure $IAPCU_o^f(x, x^f, y, \bar{\lambda})$ decreases almost monotonically in $\bar{\lambda}$, and on average, $IAPCU_o(x, x^f, y, \bar{\lambda})$ is first smaller under NC than under C and then the converse. Second, $IAPCU_o^f(x, x^f, y, \bar{\lambda})$ becomes stationary after $\bar{\lambda}$ reaches the value of 3 under C and the value of 2 under NC. Third, whereas $IAPCU_o(x, x^f, y, \bar{\lambda}) \gtreqless APCU_o(x, x^f, y, \bar{\lambda})$, for the majority

Table 5. Descriptive Statistics of Biased Industry Plant Capacity Utilization

	$IADF_o^f(x^f, y, \bar{\lambda})$									
	$\bar{\lambda} = 0.5$	$\bar{\lambda} = 1$	$\bar{\lambda} = 1.5$	$\bar{\lambda} = 2$	$\bar{\lambda} = 2.5$	$\bar{\lambda} = 3$	$\bar{\lambda} = 3.5$	$\bar{\lambda} = 4$	$\bar{\lambda} = 4.5$	$\bar{\lambda} = 5$
Convex										
Average	12.092	12.973	13.366	13.576	13.644	13.655	13.655	13.655	13.655	13.655
SD	77.335	77.236	77.181	77.148	77.139	77.137	77.137	77.137	77.137	77.137
Minimum	0.010	0.010	0.318	0.318	0.918	1.000	1.000	1.000	1.000	1.000
Maximum	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250
$ADF_o^f(.) < IADF_o^f(.)$	73	112	128	145	130	119	110	99	89	82
$ADF_o^f(.) = IADF_o^f(.)$	0	2	7	20	38	73	82	93	103	110
$ADF_o^f(.) > IADF_o^f(.)$	82	78	57	27	24	0	0	0	0	0
Nonconvex										
Average	11.547	12.225	12.450	12.541	12.541	12.541	12.541	12.541	12.541	12.541
SD	77.357	77.270	77.240	77.226	77.226	77.226	77.226	77.226	77.226	77.226
Minimum	0.010	0.080	0.318	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Maximum	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250	884.250
$ADF_o^f(.) < IADF_o^f(.)$	60	125	103	107	99	85	77	72	62	59
$ADF_o^f(.) = IADF_o^f(.)$	5	29	60	85	93	107	115	120	130	133
$ADF_o^f(.) > IADF_o^f(.)$	86	38	29	0	0	0	0	0	0	0

Table 6. Descriptive Statistics of Unbiased Industry Plant Capacity Utilization

	$IAPCU_o(x, x^f, y, \bar{\lambda})$									
	$\bar{\lambda} = 0.5$	$\bar{\lambda} = 1$	$\bar{\lambda} = 1.5$	$\bar{\lambda} = 2$	$\bar{\lambda} = 2.5$	$\bar{\lambda} = 3$	$\bar{\lambda} = 3.5$	$\bar{\lambda} = 4$	$\bar{\lambda} = 4.5$	$\bar{\lambda} = 5$
Convex										
Average	12.698	5.761	0.746	0.591	0.526	0.522	0.522	0.522	0.522	0.522
SD	23.797	19.196	0.624	0.473	0.273	0.269	0.269	0.269	0.269	0.269
Minimum	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016
Maximum	98.250	98.250	3.150	3.150	1.090	1.000	1.000	1.000	1.000	1.000
$APCU_o^f(.) < IAPCU_o^f(.)$	82	78	57	27	24	0	0	0	0	0
$APCU_o^f(.) = IAPCU_o^f(.)$	0	2	7	20	38	73	82	93	103	110
$APCU_o^f(.) > IAPCU_o^f(.)$	73	112	128	145	130	119	110	99	89	82
Nonconvex										
Average	10.792	0.811	0.673	0.553	0.553	0.553	0.553	0.553	0.553	0.553
SD	20.264	1.006	0.543	0.304	0.304	0.304	0.304	0.304	0.304	0.304
Minimum	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
Maximum	98.250	12.543	3.150	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$APCU_o^f(.) < IAPCU_o^f(.)$	86	38	29	0	0	0	0	0	0	0
$APCU_o^f(.) = IAPCU_o^f(.)$	5	29	60	85	93	107	115	120	130	133
$APCU_o^f(.) > IAPCU_o^f(.)$	60	125	103	107	99	85	77	72	62	59

of observations, we find that $APCU_o(x, x^f, y, \bar{\lambda}) > IAPCU_o(x, x^f, y, \bar{\lambda})$ until $\bar{\lambda}$ reaches the value of 4 under C and only 2.5 under NC and $APCU_o(x, x^f, y, \bar{\lambda}) = IAPCU_o(x, x^f, y, \bar{\lambda})$ afterward for the majority of observations. Furthermore, $APCU_o(x, x^f, y, \bar{\lambda}) < IAPCU_o(x, x^f, y, \bar{\lambda})$ becomes 0 when $IAPCU_o(x, x^f, y, \bar{\lambda})$ becomes stationary.

By solving the models in Definition 9, we obtain two critical points: under C, $L^{I,C} = 0.1199$ and $U^{I,C} = 2.7516$, and under NC, $L^{I,NC} = 0.1199$ and $U^{I,NC} = 1.9947$. Regarding these points, first, although the lower bound is identical under C and NC, the upper bound is substantially lower under NC than under C. Second, according to Proposition 4, for $\bar{\lambda} \geq 2.7516$ in the C case and $\bar{\lambda} \geq 1.9947$ in the NC case, we have $IADF_o^f(x_p^f, y_p, \bar{\lambda}) = DF_o^f(x_p^f, y_p)$ and $IAPCU_o(x_p, x_p^f, y_p, \bar{\lambda}) = PCU_o(x_p, x_p^f, y_p)$. Thus, in Tables 5 and 6, the last five columns in the C case and the last seven columns in the NC case contain identical results. Third, it makes no sense to compare these two critical points L^I and U^I with, for example, the averages of the corresponding points in the firm models L_p and U_p.

Instead, Table 7 reports the degree of increase of aggregate variable inputs, such that all units obtain the maximum of the standard plant capacity utilization measure $DF_o^f(x_p^f, y_p)$ from both the perspective of the firm and the industry in both the C and NC cases. The second column shows the sum of observed variable inputs. The sum of needed variable inputs with the firm-level model (9) under C and NC appears in the third and fifth columns, respectively. The fourth and sixth columns present the sum of needed variable inputs with the industry-level model (14) under C and NC, respectively. The second part of this table shows the magnification factors computed by taking the ratios of the sum of needed variable inputs to the sum of observed variable inputs under firm and industry models and under C and NC. The rows denote the two variable inputs: water and workers.

In analyzing the results in Table 7, we draw three conclusions. First, firm models need many more variable

Table 7. Amounts of Variable Inputs Across Models

		Convex		Nonconvex	
Variable inputs	$\sum_{p=1}^K x_p^v$	$\sum_{p=1}^K U_p x_p^v$	$\sum_{p=1}^K U^I x_p^v$	$\sum_{p=1}^K U_p x_p^v$	$\sum_{p=1}^K U^I x_p^v$
Billions of cubic meters of water	30,718.888	103,352.775	84,526.092	74,867.372	61,274.966
No. of workers	3,203.284	94,183.888	8,814.156	89,220.392	6,389.590

	Convex		Nonconvex	
Variable inputs	$\frac{\sum_{p=1}^K U_p x_p^v}{\sum_{p=1}^K x_p^v}$	$\frac{\sum_{p=1}^K U^I x_p^v}{\sum_{p=1}^K x_p^v}$	$\frac{\sum_{p=1}^K U_p x_p^v}{\sum_{p=1}^K x_p^v}$	$\frac{\sum_{p=1}^K U^I x_p^v}{\sum_{p=1}^K x_p^v}$
Billions of cubic meters of water	3.364	2.752	2.437	1.995
No. of workers	29.402	2.752	27.853	1.995

inputs than industry models. Second, C models need many more variable inputs than NC models. Third, whereas the industry models with an almost doubling of variable inputs under NC and an almost tripling of variable inputs under C are not necessarily incredible, the firm models with a doubling by a factor of almost 2.5 at a minimum and a 30-fold magnification at worst are clearly incredible. For the number of workers it is simply inconceivable that firms could magnify the existing numbers by a factor of 27.85 under NC and a factor of 29.40 under C.

We deduce the following overall conclusions. First, firm models necessitate unlikely numbers of variable inputs, whereas the results for industry models are not a priori strikingly unrealistic. Second, NC models involve less unrealistic numbers of variable input magnifications than C models.

Although some may put their hope in the industry models, it is crucial to remember their limitations. First, these models presuppose that there is a central authority coordinating among all firms. If firms are decentralized, this clearly is not an option. Second, the industry models are very basic. However, a more realistic industry model with additional constraints [e.g., constraints on the amounts of inefficiency that are allowed for (see Kerstens et al. 2006), putting lower and upper bounds on changes in variable inputs per firm, etc.] would lead to less spectacular results.

To provide additional empirical evidence, in Section EC.3 in the e-companion we report the empirical results for both the firm and industry levels for a data set with multiple outputs. We find that the multiple-output results are slightly less extreme than the single-output results. Clearly, these empirical illustrations make the basic point about the attainability issue of the traditional output-oriented plant capacity measure.

5. Conclusions

The output-oriented plant capacity concept has been around for at least two decades and is quite popular for empirical applications. Although it was directly inspired by the informal definition provided by Johansen (1987), the doubts of Johansen expressed with regard to the attainability of the concept have seemingly never been investigated. This paper has tried to dig deeper into this issue of attainability.

In Section 3, we formally defined both the traditional output-oriented and the rather new input-oriented plant capacity notions. Thereafter, we argued that the output-oriented plant capacity notion may well fail the notion of attainability in general, because the numbers of variable inputs required to reach the maximum capacity outputs may simply not be available. This issue does not seem to be a problem for the input-oriented plant capacity concept. Consequently, we defined a new attainable output-oriented plant capacity notion that incorporates either firm or industry constraints on the availability of variable inputs. It is up to the researcher to determine plausible values limiting the upward scaling of variable inputs.

Using secondary data, we developed an empirical illustration in Section 4, which enabled us to draw several conclusions. First, outputs need to be magnified unreasonably to reach traditional plant capacity outputs. Second, the reason for this phenomenon is that variable inputs are supposed to be scalable at numbers that are unlikely to be available at either the firm or the industry level. Moreover, the degree of scaling that needs to be applied is ways above the fivefold increase, a case that we experimented with when defining our attainable plant capacity notion. Third, although this scaling of variable inputs is likely unreasonable, computational results on a nonconvex technology are slightly less implausible than those obtained on a traditional convex technology. Thus, nonconvexity seems to partly mitigate the extreme results associated with the traditional output-oriented plant capacity notion. Fourth, the industry model (if applicable) leads to less incredible results than the firm model.

It is clear that the traditional output-oriented plant capacity concept in general faces serious attainability problems. To continue using this output-oriented plant capacity concept, one should always at least compute the critical point U_p to verify whether the required variable inputs are likely available at the firm level. If these required variable inputs are unavailable, then the new notion of an attainable output-oriented plant capacity concept merits further attention. Furthermore, because the new input-oriented plant capacity notion does not face any attainability issues, it may constitute an alternative framework as well.

We propose three avenues for future research. First, our empirical analysis related to the attainability problem of the traditional output-oriented plant capacity concept needs further corroboration. In particular, it would be important to verify whether the attainability problem is equally serious when employing alternative estimators (e.g., stochastic frontier analysis; see Felthoven 2002). Furthermore, a major limitation is that we limited our analysis to radial efficiency measures, although it is well known that the traditional convex and especially the nonconvex technologies suffer from a great amount of unmeasured inefficiency appearing as slacks (see, e.g., De Borger et al. 1998). There are some indications that slacks may also play a substantial role in the measurement of plant capacity utilization (e.g., Dupont et al. 2002 or Vestergaard et al. 2003). Therefore, it would be useful to revisit the attainability problem using nonradial rather than radial efficiency measures.

Second, our attainable plant capacity notion could benefit from a clarification of the number by which variable inputs can reasonably be magnified (i.e., the value of $\bar{\lambda}$). Expert opinion may be one source of inspiration worth exploring. Economic considerations related to, for example, cost minimization or profit maximization may be another source of inspiration. Otherwise, it remains a conceptual alternative for the traditional output-oriented plant capacity notion, but it has little empirical bite.

Third, Kerstens et al. (2017) find that the input-oriented plant capacity notion compares well with cost-based capacity notions, whereas the output-oriented plant capacity notion performs less well in this respect. Thus, we question the extent to which the attainability issue of the traditional output-oriented plant capacity plays a role in these results. Perhaps the attainable output-oriented plant capacity would mitigate these differences: this remains an open question.

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Endnotes
[1] For example, the convex variable returns to scale technology does not satisfy inaction.

[2] In case of parametric production technologies with a single output, Färe (1984) formally defines the notions of plant capacity limiting and weakly plant capacity limiting factor combinations and provides necessary and sufficient conditions for a factor combination to be plant capacity limiting, assuming additional restrictions on the class of production functions. However, not all production functions satisfy these additional restrictions. For example, for a popular production function such as the constant elasticity of substitution with certain parameter values, no factor combination is (weakly) plant capacity limiting.

[3] Sahoo and Tone (2009) propose another input-oriented capacity notion based on the short-run technology T^f. Its eventual relationship to $PCU_i(x, x^f, y)$ remains to be explored.

[4] The idea of a kind of limited strong disposability has been pursued in the context of congestion measurement (see Briec et al. 2016).

[5] Although inaction is often phrased mathematically as $(0, 0) \in T$, the occurrence of zero outputs need not imply zero inputs. Assuming strong input disposability, $(x, 0) \in T$ for $x > 0$. Thus, the use of positive inputs is compatible with zero outputs.

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